INTEGRATED GANIT

Class IX & X
VOL- 1

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INTEGRATED
GANIT

Class IX & X
(1st Vol.)

[Arithmetic & Algebra]

This is the only Text Book on Mathematics
for Class IX & X of all affiliated School

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Foreword

The existing Syllabi of Mathematics for Classes VI to X in the Schools recognised by the Board have been running in phases since 1981. Accordingly, the text books of Mathematics for Classes VII to X are published by the Board. The text book of Mathematics for Class VI is the publication of Govt. of West Bengal . The Government, however, arranges for free distribution of the book.

Necessity had been duly felt to rearrange the contents of the erstwhile two different Mathematics books, one each for Class IX and Class X. This was particularly in consideration that the Syllabus of Mathematics as the Syllabi of all other Subjects for Madhyamic Pariksha of the Board is an integrated one for Class IX and X. Consequently, as well as in view of the volume of the book, the Mathematics text book for Classes IX and X taken together is now published in two parts - part I consisting of Algebra and Arithmetic, Part II of Geometry, Trigonometry and Mensuration.

This has been a matter of privilege to the Board that many teachers, educationists and also some students had suggested some corrections in our text books on Mathematics. Efforts have been made to incorporate the corrections, as well as to minimise certain shortcomings to uplift the standard of the books.

In this context I like to impress upon of all concerned that the books prepared by the Board are the only text books on Mathematics approved by the Board. The School authorities are, therefore, requested not to put the guardians and their wards in confusion by prescribing any other Mathematics books whatsoever as text books for Class VI to X.

I look forward for the cooperation of teachers, educationists and students also in future as in the past.

Chittaranjan Bandhopadhyay
President,
West Bengal Board of Secondary Education

77/2, Park Street,
Calcutta - 700 016,
The 17th July, 1992.

Revised Syllabus in Mathematics for Class IX & X
(Arithmetic & Algebra)

	Marks
Arithmetic	**15**

1. Ratio and Proportion. Applications to simple problems.
2. Percentage. Applications to simple problems.
3. Harder fractions.
4. Simple Interest, Applications to problems that may arise in daily life.
5. Profit and loss. Applications to problems that may arise in family and community activities.
   (Problems may be solved algebraically)

Algebra	**30**

1. Revision of previous work through exercises.
2. Formation of linear simultaneous equations with two variables to represent simple problems.
3. Solution of linear simultaneous equations containing two unknowns. (Cases of non-existence of unique solution to be explained.)
4. Graphical representation of linear equations containing one or two variables.
5. Ratio and Proportion.
6. Revision of previous work through exercises.
7. Graphical solution of problems involving one or two unknowns.
8. Problems leading to formation of simple quadratic equations with one variable which can be solved by easy factorisation, and their solutions.
9. Simple and simultaneous inequations. Determination of their solution set, use of graphs.
10. Simple quadratic surds, basic operations on them.

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ARITHMETIC

CHAPTER 1

RATIO

Suppose 7 kg of copper and 3 kg of zinc be melted together to make a particular type of brass.

Then, we can say, 10 kg of brass contains 7 kg of copper and 3 kg of zinc.

Now, what will be the weights of copper and zinc in kilograms, in 5 kg of brass of the same kind ?

We know that 5 kg is half of 10 kg. Hence the quantity of copper and zinc present in 5 kg of brass will be just half of the quantity of copper and zinc present in 10 kg of brass.

$\because$ 10 kg of brass contains 7 kg of copper.

$\therefore$ 5 kg of brass contains $(7 \div 2)$ kg $= 3\frac{1}{2}$ kg of copper.

and 10 kg of brass contains 3 kg of zinc.

$\therefore$ 5 kg of brass contains $(3 \div 2)$ kg $= 1\frac{1}{2}$ kg of zinc.

Again, if we take 20 kg of brass of same kind, then what will be the quantity of copper and zinc in 20 kg of brass ?

Since 20 kg is double of 10 kg, the amount of copper in 20 kg will be (7×2) kg $= 14$ kg, and the amount of zinc in 20 kg of brass will be (3×2) kg $= 6$ kg.

We find that when the amount of copper is doubled, the amount of zinc is also doubled.

So, in this particular mixture of copper and zinc, there exists a particular relation between the quantity of copper and zinc. Out of 10 parts of brass, 7 parts is copper and 3 parts is zinc. Again, out of 5 parts of brass, $3\frac{1}{2}$ parts is copper and $1\frac{1}{2}$ parts is zinc. 20 parts of brass contains 14 parts of copper and 6 parts of zinc.

This particular relation, expressed above, is written in mathematical language as,

The ratio of copper and zinc in the brass is 7 : 3.

We express the above relation as,
copper : zinc = 7 : 3
of zinc : copper = 3 : 7

The mathematical symbol of ratio is ":"

[*Remark* : At the time of expressing the ratio of weights, volumes etc. of different substances we generally mention the names of the two substances for the ratio of their measures. As in the above example, we have written zinc : copper as ratio of their respective weights. But we have to mention units, if two substances bear different units. Again, son's amount : daughter's amount, may be expressed in brief as son : daughter.]

Ratio can be expressed as 'fraction' :

Let us consider the same example of brass, where 10 parts of the brass contains 7 parts of copper and 3 parts of zinc. We can express; each of these ingredients (copper and zinc) in terms of the fractional parts of the mixture (brass).

The quantity of the first ingredient $=\frac{7}{12}$ part of the mixture.

The quantity of second ingredient $=\frac{3}{10}$ part of the mixture.

Hence, $\frac{\text{First ingredient}}{\text{Second ingredient}} = \frac{\frac{7}{10}}{\frac{3}{10}} = \frac{7}{3}$

$\therefore$ First ingredient $=\frac{7}{3}$rd of the second ingredient, and Second ingredient, $=\frac{3}{7}$ th of the first ingredient.

So in fractional form we can write $7:3=\frac{7}{3}$ and $3:7=\frac{3}{7}$.

From the above example we also find that the ratio, copper : zinc, can be expressed by any one of 7 : 3 or $3\frac{1}{2}:1\frac{1}{2}$ or 14 : 6. Hence, in terms of ratio, $7:3=3\frac{1}{2}:1\frac{1}{2}$ $=\frac{7}{2}:\frac{3}{2}=14:6$.

Thus we need two numbers to express a ratio and his ratio remains unchanged if we multiply or divide these two numbers by the same numbers.

Example 1. The length and breadth of a rectangle are 15 m and 8 m respectively. Write down the ratio of the length to the breadth of the rectangle.

Ans : length : breath = 15 : 8

[*Remarks* : It may be noted that in the above example we have used the unit 'metre' to express the length and breadth of the rectangle. But the ratio of 15 m to 8 m is only a pure number. The ratio is 15 : 8. Similarly the ratio of Rs. 15 to Rs. 8 is 15 : 8. No unit should be attached to a ratio as ratio is a pure number.

Example 2. To clean the floor of a hospital 1 litre 3 decilitres phenol is mixed with 17 litres of water. Find the ratio of the quantity of water to phenol.

17 litres = 170 decilitres.

1 litre 3 decilitres = 13 decilitres.

Hence, water : phenol = 170 : 13.

[*Remark* : In this example the quantity of water has been expressed in the unit 'litre' and the quantity of phenol has been expressed in the mixed unit, 'litre and decilitre'. In this case, we must express both the quantities in the same unit. In the above example, we have expressed both the quantities in 'decilitre'. We can also solve the above example by expressing both the quantities in the unit 'litre'.

The quantity of water = 17 litres

The quantity of phenol = 1 litre 3 decilitres = 1·3 litres

$$\text{Hence, water : phenol} = 17 : 1{\cdot}3$$
$$= 170 : 13$$

for the last step both the numbers related to the ratio have been multiplied by 10.]

You have learnt to express any number or quantity by algebraic symbol. Now suppose '*a*' kg of ammonia is mixed with '*b*' kg of potash to make a mixed fertilizer.

Then the ratio of ammonia to potash will be

ammonia : potash = $a : b$.

Some facts about ratio :

You know that the ratio of two quantities is a pure number and has no unit.

Each of the numbers by which a ratio is expressed is known as 'term'.

The first term is called 'antecedent' and the second term 'consequent'.

For example in the ratio 3 : 5, 3 is the antecedent and 5 is the consequent.

Again, in the ratio c : d, *c* is the antecedent and *d* is the consequent.

You have seen that ratio can be expressed as fraction, for example, $3 : 5 = \frac{3}{5}$, $15 : 8 = \frac{15}{8}$ and $p : q = \frac{p}{q}$. Hence, all the rules applicable to fractions are applicable to ratio also.

(a) A fraction remains unchanged if the numerator and the denominator of the fraction be multiplied or divided by the same number.

Similarly, the ratio of two numbers remains unchanged if the antecedent and the consequent be multiplied or divided by the same number.

For example, $3 : 5 = (3 \times 2) : (5 \times 2) = 6 : 10$

and $8 : 12 = (8 \div 4) : (12 \div 4) = 2 : 3$

and $a : b = (a \times c) : (b \times c) = ac : bc$

You have known about this fact from your previous knowledge.

(b) As a fraction can be expressed in its lowest form, a ratio can also be expressed in lowest terms.

For examle, $15 : 20 = (15 \div 5) : (20 \div 5) = 3 : 4$

and $18 : 12 = (18 \div 6) : (12 \div 6) = 3 : 2$

and $ab : bd = (ad \div d) : (bd \div d) = a : b$

Inverse ratio :

You know that when numerator and demoninator of a fraction are interchanged we get the reciprocal of the fraction.

For example, reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$.

Similarly, if in a ratio antecedent and consequent are interchanged, we get a ratio, which is called 'inverse ratio' with respect to the original ratio.

Now in the ratio 3 : 5, 3 is the antecedent and 5 is the consequent.

Hence, the inverse ratio of 3 : 5 will be 5 : 3. Here antecedent is 5 and consequent is 3.

Similarly, the inverse ratio of $c : b$ is $b : c$.

Inverse ratio can be explained in terms of the rules of fractions. $3 : 5 = \frac{3}{5}$, the inverse ratio $= \frac{1}{\frac{3}{5}} = 1 \times \frac{5}{3} = \frac{5}{3} = 5 : 3$.

Mixed or Compound ratio :

If a new ratio be formed by the continued product of antecedents and continued product of consequents, of two of more simple ratios, then new ratio is called a mixed or compound ratio.

For example, we have three ratios 3 : 2, 4 : 5, 2 : 7.

The continued product of all antecedents of the above simple ratios is $3 \times 4 \times 2 = 24$.

The continued product of all the consequents of the above simple ratios is $2 \times 5 \times 7 = 70$.

Hence, the compound ratio is 24 : 70 = 12 : 35

Similarly, the compound ratio formed from the simple ratios $a : b$ and $c : d$ is $(a \times c) : (b \times d) = ac : bd$.

Example 3. Find the inverse ratios of the following ratios.

(i) 7 : 12 (ii) 13 : 9 (iii) 16 : 7 (iv) 25 : 43

Ans : (i) 12 : 7 (ii) 9 : 13 (iii) 7 : 16 (iv) 43 : 25

Example 4. Express the following ratios in lowest terms.

(i) 8 : 14 (ii) 42 : 21 (iii) 45 : 18 (iv) 20 : 65

Ans. (i) 4 : 7 (ii) 2 : 1 (iii) 5 : 2 (iv) 4 : 13

Absolute character of ratio :

Though the ratio expresses the comparison between two quantities of same kind, yet, it is a relation having no units attached to it. The ratio is not at all related to the kind and nature of the quantities. Hence, the ratio of a pair of quantities of different kinds may be the same.

For example, (*i*) the ratio of two pipes of lengths 8 m and 12 m is 8 : 12 = 2 : 3

(*ii*) A train has been running for 8 hours and another train running for 12 hours. The ratio of the time of running of two trains is 8 : 12 = 2 : 3.

In (*i*) the ratio 2 : 3 is the ratio of lengths and in (*ii*) the ratio 2 : 3 is the ratio of time durations.

Again, the ratios of pairs of quantities of the same kind but of different magnitudes may be same.

For example, (iii) two packets contain 4 kg and 6 kg of sugar respectively. The ratio of their weights is 4 : 6 = 2 : 3.

(*iv*) Again two bags contain 28 kg and 42 kg of coal. The ratio of their weights is 28 : 42 = 2 : 3.

In these two examples the magitudes of the quantities differ from each other but their respective ratios are the same.

Now in the example (i) the antecedent of the ratio 2 : 3 is 2. This number '2' does not indicate truely the length of the first pipe. Similarly, the consequent 3 of the same ratio 2 : 3, does not indicate the true length of the second pipe. But if along with the ratio, the true length of one of the pipes is given, then we can find the true length of the other pipe.

Note the following example :

Example 5. The ratio of ages of father and son is 7 : 2. If the age of son is 12 years, what will be the age of father ?

Given that,

Age of father : Age of son = 7 : 2
And the age of son is 12 years

Now we know that,

if First term : Second term = 7 : 2

then First term = Second term $\times \frac{7}{2}$

Hence, the age of father = age of son $\times \frac{7}{2} = (\frac{7}{2} \times 42)$ years

[*Remark* : Although we cannot find the magnitude of two quantities from their ratio, but we can say that the number by which antecedent is to be multiplied to get the first quantity, the same number is to be multiplied with the consequent to get the second quantity. For example, if the ratio of two quantities is $a : b$, then those two quantities may be assumed as ax and bx respectively. Afterwards, if x is found out, the magnitude of both the quantities will be known.

For instance ; age of father : age of son = 7 : 2

∴ let age of father be $7x$ years and that of son be $2x$ years.

∴ By condition of the problem $2x = 12$ or $x = 6$

∴ age of father = (7×6) years = 42 years.]

Ratio of more than two quantities :

In the first example of this chapter you have seen that brass contains copper and zinc. But there are many substances which are made of more than two ingredients and there exists a relation among the quantities of the different ingredients.

Such as, a special kind of german silver may be had by mixing 8 kg of copper, 7 kg of lead and 5 kg of nickel.

In this case, we can express the weights of the different ingredients of german silver in the form of a ratio in the following way :

copper : lead : nickel = 8 : 7 : 5

Similarly, if the length, breadth and height of a room be respectively 4·5 m, 3·5 m and 3 m, we can write,

length : breadth : height = 4·5 : 3·5 : 3
= 9 : 7 : 6

Again, if the ages of Ram, Shyam, Jadu and Madhu be *a*, *b*, *c*, and d years respectively, then we can write,

Ram : Shyam : Jadu : Madhu = $a : b : c : d$

Exercise 1

1. Express the following pairs of numbers in ratio :

(i) 12 kg and 8 kg, (ii) 45 m and 54 m, (iii) 5 hours and 13 hours, (iv) 1 m 4 dcm and 7 dcm (v) 4 hours 15 minutes and 5 hours, (vi) 7 litres 2 decilitres and 9 litres, (vii) 2 months 5 days and 1 month 9 days.

2. What is the ratio of 60 paisa to Rs. 4 and 80 paisa ?

3. Heights of Debashis and Debraj are 1 m 74 cm and 90 cm respectively. What is the ratio of their heights ?

4.(i) In the years 1979 and 1980, 19,838 and 25,942 numbers of different kinds of books were published. What is the ratio of the published books in those two years ?

(ii) Out of all the books published in the year 1980, 2,475 books are for children and 440 books are text books. What is the ratio of these two kinds of books ?

5. Find the inverse ratio of the following ratios :

(i) 8 : 5, (ii) 7 : 12, (iii) 22 : 13, (iv) 25 : 21, (v) 48 : 83, (vi) 125 : 12, (vii) 7 : 118, (viii) 129 : 225.

6. Express the following ratios in lowest terms :

(i) 8 : 6, (ii) 3 : 6, (iii) 26 : 13, (iv) 25 : 40, (v) 65 : 91, (vi) 111 : 27, (vii) 120 : 135, (viii) 119 : 170, (ix) $\frac{2}{9} : \frac{5}{6}$, (x) $\frac{1}{5} : \frac{3}{7}$, (xi) $\frac{7}{5} : 3$, (xii) $\frac{5}{8} : 1\frac{1}{16}$, (xiii) $7\frac{1}{2} : 8\frac{1}{3}$.

7. In the following table find out the missing terms in the places with asterick (*) mark.

Ratio	Antecedent	Consequent
6 : 8	$6 \times 3 = 18$	$8 \times 3 = 24$
5 : 7	25	*
15 : 19	*	76
22 : 7	$1\frac{4}{7}$	*
65 : 22	*	2·2
3 : 7	*	2·24
14 : 19	2·8	*
$x : y$	$\frac{x}{y}$	*
$p : q$	pq	*

8. A compound fertilizer is made of 6 kg of ammonia, 8 kg of potash and 20 kg of cowdung. Find the ratio of the different ingredients in that compound fertilizer.

9. From Sunday to Wednesday you have worked out 9, 6, 12 and 15 sums respectively. Write down the ratio of workedout sums on different days.

10. Express the following ratios as the ratio of two whole numbers :

(i) $\frac{1}{4} : \frac{1}{6}$, (ii) $\frac{4}{37} : \frac{11}{74}$, (iii) 7·7 : 6·3.

11. The ratio of the ages of Sipra and Moyna is 7 : 4. If the age of Sipra is 35 years, find the age of Moyna.

12. The ratio of number of students of class IX of a school in Section A to that in section B is 6 : 5 and the number of students on Section B is 35. What is the number of students in Section A ?

13. The ratio of the prices of two books is 3 : 5. If the price of the first book be Rs. 4·80, find the price of the second book.

14. The ratio of the circumference of the diameter of a circle is 22 : 7. Find the circumference of the circle whose diameter is 4m 2 dcm.

15. The ratio of the speed of a passenger train to that of a goods train is 4 : 3. If the goods train covers a distance of 72 km in 4 hours, then what is the distance travelled by the passenger train in 3 hours ?

16. Out of the three balls of red, black and white colour, the ratio of the weights of red and black balls is 5 : 6 and that of black and white is 5 : 4. If the weight of black ball is 180 gms, find the ratio of weights of other two balls.

17. A cyclist, after cycling a distance of 60 km on the second day, finds that the ratio of the distances covered by him on the first two days is 4 : 5. If he travels a distance of 48 km in the third day, find the ratio of the distances travelled on the first and the third days.

18. The ratio of the prices of two houses is 4 : 3 and the price of the second house is Rs. 42,000. Had the price of the first house been Rs. 7,000 more, what would have been the ratio of their prices ?

19. If a piece of length 15m be cut from a bamboo pole, the ratio of the lengths of the two portions is 3 : 1. Find separately the ratios of the lengths of the two portions of the pole to its original length.

20. The bus fares of three cities in terms of distances are given below. Find the ratio of the bus fares of the three cities for the same distance.

	(i) 10 km	(ii) 15 km	(iii) 20 km
Bombay	0·75	0·95	1·15
Calcutta	0·40	0·50	0·50
Madras.	0·70	0·85	0·95

Proportion

Suppose, two friends, Amal and Murshid have bought 4 kg and 6 kg of rice respectively at Rs. 4 per kg.

Now, Amal has paid Rs. 4 × 4 = Rs. 16 for 4 kg of rice and Murshid has paid Rs. 4 × 6 = Rs. 24 for 6 kg of rice. Now if you are asked

(i) to compare the quantity of rice they have bought, and (ii) to compare the price they have paid for them you will say,

quantity of rice bought by Amal : quantity of rice bought by Murshid = 4 : 6

and, price paid by Amal : price paid by Murshid = 16 : 24.

If we express these two ratios in lowest terms, then 4 : 6 = 2 : 3 and 16 : 24 = 2 : 3

Hence 4 : 6 = 16 : 24.

In these case, we say that the terms 4,6,16 and 24 are in proportion.

This is expressed as

4 : 6 = 16 : 24 or, 4 : 6 : : 16 : 24.

Let us consider a few other examples on proportion.

Example 1. At uniform speed a train runs 140 km in 4 hours and 210 km in 6 hours.

Now the ratio of time is, 4 : 6 = 2 : 3 and the ratio of distances covered by the train, 140 : 210 = 2 : 3.

Hence, ratio of time = ratio of distance travelled

i.e., 4 : 6 = 140 : 210.

∴ the four terms are in proportion, as the relation between the time taken and the distance travelled are proportional.

N.B. In the above two examples, we find that there is a direct relation between the different quantities. When the quantity of rice is increased, the total cost is increased and when the time is increased, the distance covered also increases.

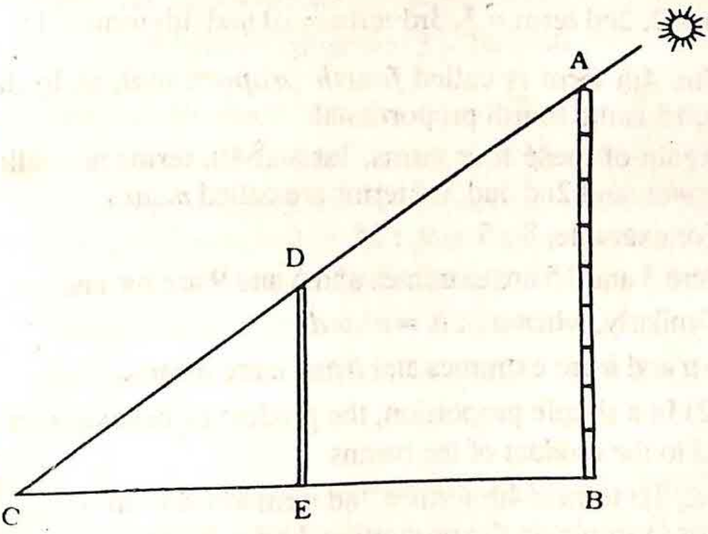
When this type of direct relation exists between two ratios which are in proportion, then the proportion is known as simple proportion.

How can we express this relation in algebraic symbols ?

Suppose at a fixed market price of rice Amal buys c kg of rice for Rs. a and Murshid buys d kg of rice for Rs. b. Then a, b, c, d are in porportion, i.e., $a : b = c : d$.

$$\text{or, } \frac{a}{b} = \frac{c}{d}$$

Geometrical interpretation of proportion :



In this fig., AB is a bamboo-pole and BC is its shadow, DE is a stick and EC is its shadow.

Using the scale : 1 m = 1 cm, it is found that

$AB = 5\text{m}$ $BC = 7{\cdot}5\text{m}$

$DE = 2\text{m}$ $EC = 3\text{m}$

$\therefore$ length of bamboo pole : length of stick

$= AB : DE = 5 : 2$

length of shadow of bamboo pole : length of shadow of stick

$$= BC : EC = 7{\cdot}5 : 3 = 5 : 2$$

$\therefore$ AB, DE, BC, EC are in proportion

i.e., $AB : DE = BC : EC$

or, $\dfrac{AB}{DE} = \dfrac{BC}{EC}$

Some fundamental properties of proportion :

(1) The terms of a proportion are called 1st term, 2nd term, 3rd term and 4th term respectively.

For example, 2 : 3 = 10 : 15, in this case, the terms of the proportion are respectively 2,3,10,15. Of them, 1st term = 2, 2nd term = 3, 3rd term = 10 and 4th term = 15.

The 4th term is called *fourth proportional*, as in this case, 15 is the fourth proportional.

Again of these four terms, 1st and 4th terms are called *extremes*, and 2nd and 3rd terms are called *means*.

For example, 3 : 5 = 9 : 15,

here 3 and 15 are extremes and 5 and 9 are means.

Similarly, when $a : b = c : d$

then a and b are extremes and b and c are means.

(2) In a simple proportion, the product of the extremes is equal to the product of the means

i.e., 1st term × 4th term = 2nd term × 3rd term

For example, in the proportion 3 : 5 = 9 : 15

1st term × 4th term = $3 \times 15 = 45$

2nd term × 3rd term = $5 \times 9 = 45$

Thus both the products are equal to each other.

So we say, if $a : b = c : d$, then $ad = bc$.

In class VII while solving equation you have used the method of cross-multiplication. Applying the method of cross-multiplication, we have, $a : b = c : d$

or, $\frac{a}{b} = \frac{c}{d}$ or, $ad = bc$.

(a) With the help of this formula we can verify whether two ratios are proportional or not,

(i) Verify if the terms of the ratios 3 : 7 and 15 : 35 are proportional.

Here, 1st term × 4th term = $3 \times 35 = 105$

2nd term × 3rd term = $7 \times 15 = 105$

Since the two products are equal, the four terms are proportional, i.e., $3 : 7 = 15 : 35$.

(ii) Verify if the terms of the ratios 4 : 3 and 10 : 16 are proportional.

Here, 1st term × 4th term = $4 \times 16 = 64$

2nd term × 3rd term = $3 \times 10 = 30$

The two products are not equal. Therefore, the four terms are not proportional, i.e., $4 : 3 \neq 10 : 16$.

(b) Four numbers are in proportion, if the product of any two of these numbers is equal to that of the other two and then proportions can be formed by those four numbers.

Let us consider the numbers 7, 5, 10 and 14.

We see that, $7 \times 5 \neq 10 \times 14$ and $7 \times 14 \neq 5 \times 10$, but $7 \times 10 = 5 \times 14$.

In this case, with the numbers 5, 7, 10 and 14 following four porportions are possible :

(i) $5 : 7 = 10 : 14$ (ii) $7 : 5 = 14 : 10$

(iii) $5 : 10 = 7 : 14$ (iv) $10 : 5 = 14 : 7$

So we see that of the four numbers if the product of any two be equal to that of the other two, then the four numbers are proportional.

Let us verify this algebraically :

Let, $ad = bc$.

(i) Dividing both sides by bd, we get,

$$\frac{ad}{bd} = \frac{bc}{bd} \text{ or, } \frac{a}{b} = \frac{c}{d}$$

i.e., $a : b :: c : d$ ... (1)

Thus we obtain the 1st proportion.

(ii) Dividing both sides by cd, we get,

$$\frac{ad}{cd} = \frac{bc}{cd} \text{ or, } \frac{a}{c} = \frac{b}{d}$$

i.e. $a : c :: b : d$ ... (2)

This is the 2nd proportion.

(iii) Dividing both sides by *ab*, we get,

$$\frac{ad}{ab} = \frac{bc}{ab} \text{ or, } \frac{d}{b} = \frac{c}{a}$$

i.e., $d : b :: c : a$

This is the 3rd proportion. ... (3)

(iv) Again dividing both sides by *ac*, we get,

$$\frac{ad}{ac} = \frac{bc}{ac} \text{ or, } \frac{d}{c} = \frac{b}{a}$$

i.e., $d : c :: b : a$ ... (4)

This is the 4th proportion.

It is not possible to obtain more than four proportions. The reason is simple. To obtain a desired proportion both sides should be divided by the product of two numbers out of a, b, c, d which should contain either a or d. But, there are only four such products and those are ab, ac, bd and cd.

We can now say something more about proportion.

Of the above four proportions if any one is true, then the other three are also true.

Now let us compare the 1st and 2nd proportion.

$a : b :: c : d$ ... (1)

$a : c :: b : d$ ... (2)

We see that in any proportion the 2nd and 3rd terms or *the two middle terms can be interchanged,* i.e., if a given proportion is true then new proportion obtained by interchanging its 2nd and 3rd terms would also be true.

Similarly, by comparing the first and the third proportions we see that the *two extremes of the proportion can be interchanged.*

Again, comparing the first and the fourth proportions, we see, that for any proportion these two operations can be done simultaneously i.e., *the two means and two extremes can be interchanged between themselves.*

(c) In a proportion, if out of the four terms three terms are known, then the fourth term can be found out. This operation is known as finding the fourth proportional.

Suppose, in a proportion the first three terms, are 5, 8, 35 respectively but the fourth term is not known and we are to find it.

Let x be the fourth term, then the proportion will be

$$5 : 8 = 35 : x$$

You have seen,

1st term × 4th term = 2nd term × 3rd term

$$\therefore 5 \times x = 8 \times 35$$

Again, according to the rule of solving simple equation,

$$5x = 8 \times 35$$

$$\therefore x = \frac{8 \times 35}{5} = 56$$

Hence the fourth term is 56.

Form this we may obtain the following formula,

$$\text{4th term} = \frac{\text{2nd term} \times \text{3rd term}}{\text{1st term}}$$

(3) If two ratios are equal then their inverse ratios will also be equal, e.g.,

If $3 : 5 = 9 : 15$, then $5 : 3 = 15 : 9$

Similarly, if $a : b = c : d$, then, $b : a = d : c$

(4) If all the terms of a simple proportion be multiplied or divided by the same number excepting zero, the proportion remains unchanged, since due to this multiplication and division the value of the ratios of the proportion does not change.

(i) $8 : 12 = 10 : 15$

Multiplying each term of the above proportion by 2, we get,

$(8 \times 2), (12 \times 2), (10 \times 2), (15 \times 2)$

i.e., 16, 24, 20 and 30

If we verify with the help of rule 2(a) (i) we find that

$16 : 24 = 20 : 30.$

(ii) $6 : 9 = 12 : 18$

Dividing each term of the above proportior by 3, we get,

$(6 \div 3) : (9 \div 3)$ and $(12 \div 3) : (18 \div 3)$

i.e., $2 : 3$ and $4 : 6$

Verifying by the above rule we find that $2 : 3 = 4 : 6$.

Form the above examples we get the following algebraic rule :

(i) If we multiply each term of $a : b = c : d$ by p, then we have, $ap : bp = cp : dp$, that is, if a,b,c,d are proportional, then pa, pb, pc, pd are proportional.

(ii) If we divide each term of $e : f = g : h$ by q, then we have, $\frac{e}{q} : \frac{f}{q} : \frac{g}{q} : \frac{h}{q}$, that is if e,f,g,h are in proportion, then $\frac{e}{q}, \frac{f}{q}, \frac{g}{q}, \frac{h}{q}$ are in proportion.

Corollary : If in any proportion, if all the terms have a common factor, then they can be expressed in the lowest form by formula (i) and if in any proportion, one or more terms are fractions then they can be converted into whole numbers with help of this rule.

For example :

(i) $8 : 10 :: 16 : 20$ can be written as $4 : 5 :: 8 : 10$ (Dividing all terms by common factor)

(ii) $\frac{1}{2} : 3 = 2 : 12$,

Here if we multiply $\frac{1}{2}$ by 2, we get a whole number.

$\therefore$ Now multiplying all the terms by 2, we get,

$$\left(\frac{1}{2} \times 2\right) : (3 \times 2) = (2 \times 2) : (12 \times 2)$$

$$\text{i.e., } 1 : 6 = 4 : 24$$

(iii) $\frac{3}{4} : 2 = \frac{3}{2} : 4$,

here to convert, $\frac{3}{4}$ and $\frac{3}{2}$ to whole numbers they should be multiplied by 4.

Now multiplying all the terms by 4, we get,

$$\left(\frac{3}{4} \times 4\right) : (2 \times 4) = \left(\frac{3}{2} \times 4\right) : (4 \times 4)$$

$$\text{i.e., } 3 : 8 = 6 : 16$$

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Inverse Proportion :

25 workmen take 12 days to repair a road. But if 30 workmen are employed, the same work is done in 10 days only.

Here the ratio of the number of workmen is $25 : 30 = 5 : 6$ and the ratio of number of days is $12 : 10 = 6 : 5$.

Here each ratio is the inverse of the other.

If we want to obtain a proportion with the help of these ratios, then direct ratio of one is to be equated with the inverse ratio of the other.

i.e., $25 : 30 = 10 : 12$ (i.e, inverse ratio of $12 : 10$)

Again, $12 : 10 = 30 : 25$ (i.e., inverse ratio of $25 : 30$)

Now the question is, why is it so ?

The answer is, here the two quantities, i.e., the number of workmen and the number of days are inversely related i.e., if the number of workmen increases, the number of days decreases.

Similarly, if two inversely related quantities are considered and the ratio of the different values of one quantity is equal to the inverse ratio of the corresponding values of the other quantity, then they are known as *inverse proportion.*

Continued Proportion :

Again, you know if the prices of 3 kg of wheat and 6 kg of wheat are Rs. 6 and Rs. 12 respectively, then,

$$3 : 6 = 6 : 12$$

$$\text{Or, } \frac{3}{6} = \frac{6}{12}$$

Here it can be noted that the consequent of the first ratio and the antecedent of the second ratio are same in magnitude. In this case, the numbers 3, 6, 12 are said to be in continued proportion.

If a, b, c are in continued proportion, then $\frac{a}{b} = \frac{b}{c}$

i.e., $a : b = b : c$

A point is to be noted carefully :

If three numbers are in continued proportion, then the second number is called the *mean proportional* and the first and the third numbers are called the *first* and the *third* proportional, repectively. So if a, b, c are in continued proportion, then b is the mean-proportional and a and c are respectively the first and third proportional.

From rule 2, you have seen.

1st term × 4th term = 2nd term × 3rd term.

For continued proportion, the rule is,

1st Proportional × 3rd Proportional = mean proportional × mean proportional

i.e., 1st proportional × 3rd proportional = (mean proportional $)^2$

Two more corollaries may be obtained :

(i) first proportional $= \dfrac{(\text{mean Proportional})^2}{\text{3rd Proportional}}$

(ii) third Proportional $= \dfrac{(\text{mean Proportional})^2}{\text{1st Proportional}}$

Algebraically, if $a : b = b : c$, then $ac = b^2$,

$$a = \frac{b^2}{c} \text{ and } c = \frac{b^2}{a}$$

Example 1. Is it possible to form a proportion with the four numbers 2, 3, 8, 12 ? If possible, then show it.

You know, in a proportion, the product of the two extremes is equal to the product of the two means.

Accordingly, taking 2 and 12 as the extremes and 3 and 8 as means, we get,

$$2 \times 12 = 3 \times 8$$

Therefore, it is possible to form proportions with these numbers and the proportions are as follows :

$$2 : 3 = 8 : 12$$

or, $2 : 8 = 3 : 12$

Again by taking 3 and 8 as the extremes and 2 and 12 as the means, we can have the following proportions :

$$3 : 2 = 12 : 8$$

or, $8 : 2 = 12 : 3$

N. B.: Apply formula 2(a) (ii) and note the result.

Example 2. Find the fourth proportional of 4, 8, 6.

Let x be the fourth proportional

$$\therefore 4 : 8 = 6 : x$$

You know, 1st term × 4th term = 2nd term × 3rd term

$\therefore 4x = 8 \times 6$

$\therefore x = \dfrac{8 \times 6}{4} = 12$

$\therefore$ the required fourth proportional is 12.

Example 3. Different quantities of rice and their prices are in proportion. What is the price of 9 kg of rice if the price of 7 kg of it is Rs. 29·40 ?

Suppose x is the required price of rice.

the ratio of the quantities of rice = ratio of the prices of rice.

$$\therefore 7 : 9 = 29{\cdot}40 : x$$

$$\therefore x = \frac{29{\cdot}40 \times 9}{7} = 37{\cdot}80$$

$\therefore$ the required price = Rs. 37·80.

Example 4. In a continued proportion the first and the third proportional are 18 and 32. Obtain the mean proportional.

Let the mean proportional be x.

You know that, 1st Proportional × 3rd Proportional = (mean Proportional) 2

$$\therefore 18 \times 32 = x^2$$

$$\therefore x = \pm\sqrt{18 \times 32} = \pm\ 24$$

So the mean proportionals are $\pm$ 24.

Exercise 2

1. Indicate, which of the following are in proportion and form the possible proportions :

(i) 8, 10, 16 and 20 (ii) 30, 24, 5 and 4

(iii) 6, 7, 25, and 35 (iv) 4, 10, 12 and 30

(v) $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{16}$ and $\frac{1}{32}$.

2. Find the fourth proportional of the following numbers :

(i) 5, 10, 8 (ii) 12, 16, 18 (iii) $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{9}$

(iv) 2, $4\frac{1}{2}$, 4 (v) 3, 4·5, 8 (vi) 1·2, 2·4, 0·02

(vii) 2·1, 0·21, 8

3. Find the missing terms denoted by astericks:

(i) $12 : 16 :: * : 20$ (ii) $4 : * :: 9 : 18$

(iii) $2{\cdot}6 : 1{\cdot}3 :: 1{\cdot}1 : *$ (iv) $\frac{1}{3} : \frac{1}{5} :: * : \frac{3}{10}$

4. First two of three numbers in continued proportion are given ; form the proportion.

(i) 4, 8 (ii) 9, 12 (iii) $5\frac{1}{4}$, 7 (iv) $\frac{2}{3}, \frac{4}{9}$ (v) 1·6, 2·8

5. Of three numbers in continued proportion, the extremes are given; form the proportion :

(i) 5, 45 (ii) 25, 81 (iii) $\frac{4}{9}, \frac{25}{49}$ (iv) $\frac{7}{3}, \frac{16}{21}$

(v) 1·5, 13·5 (vi) $1\frac{4}{7}, 6\frac{2}{7}$

6. We know that 8, 24, 5, 15 are proportional ; without actual calculation, say whether the following are proportional or not :

(i) $\frac{1}{8}, \frac{1}{24}, \frac{1}{5}, \frac{1}{15}$ (ii) 16, 48, 10, 30

(iii) $\frac{8}{3}, 8, \frac{5}{3}, 5$

7. The ratio of 65 to a number is equal to the ratio of 13 to 6. Find the number.

8. Find a number such that the ratio of $\frac{3}{4}$ to that number is equal to the ratio of that number to $\frac{27}{64}$.

9. If $A : B = 2 : 3$ and $B : C = 4 : 5$, find $A : B : C$.

10. The ratio of the weights of equal volumes of steel and brass is 36 : 45.Find the weight of 1 cc of steel if 1 cc of brass weighs 8·4 gms.

11. In a factory producing machine parts, the ratio of the number of parts produced per day to the number of workers employed is 5 : 2.

(i) How many workers, are to be employed for the production of 125 machine parts per day.

(ii) How many machine parts will be produced per day, if 22 workers are employed.

12. In a particular blend, the ratio of Assam tea and Derjeeling tea is 7 : 2.

(i) How much Assam tea is to be mixed with 3 kg of Darjeeling tea and what is the total quantity of the blended tea ?

(ii) Again, if the price of Assam tea is Rs. 12 per kg. and the ratio of the prices of Assam and Darjeeing tea be 2 : 7, then calculate the price of the blended tea.

13. A farmer has 40 quintals of cowdung fertilizer. He wants to produce a mixed fertilizer by mixing ammonia and potash with it in the ratio, ammonia : cowdung : potash = 2 : 5 : 3. Calculate the amount of ammonia and potash he should buy.

14. In a library, the ratio of the number of story books to other books is 4 : 3 and the total number of story books is 1248. When some more story books were bought, the ratio becomes 11 : 6. Calculate the number of story books bought.

15. A train started from Calcutta for Madhupur and another train started from Madhupur for Calcutta at the same time. The two trains reached there destinations in 1 hour and 4 hours respectively after they meet each other. Show that the ratio of the speed of the two trains was 2 : 1.

The rule of Three

So far you have solved different types of problems applying unitary method. You now observe, how these problems can be solved more easily and briefly using the idea of 'proportion'.

You know the fourth term of a proportion can be found out if other three terms are known. As the fourth term of a proportion can be found when three terms of the proportion are given, this method is known as 'Rule of Three'. The following examples will clarify this method.

Examples 1. 8 books on mathematics cost Rs. 56·00. How much will be required to purchase 11 books of the same kind ?

The problem may be posed as,

No. of books	Price in rupees
8	56
11	required value

If you analyse the problem you will realise that with the increase of the number of books, the price of the books will also increase. That is, the ratio of the number of books is directly related with the ratio of the price of books. So we can write,

ratio of the price of books

= ratio of the number of books.

Let the price of 11 books be Rs x. Then the proportion will be

$$56 : x = 8 : 11$$

$$\text{or } \frac{56}{x} = \frac{8}{11}$$

$$\text{or } 8x = 56 \times 11$$

$$\text{or } x = \frac{56 \times 11}{8} = 77$$

i.e., Rs. 77·00 will be required to purchase 11 books.

Example 2. If 15 workers can make an instrument in 8 days, then in how many days can 20 workers make such an instrument ?

The problem may be posed as

No.of workers	No. of days worked
15	8
20	?

Analysing the problem you will find that the rate of increase of the number of workers will be same as the rate of decrease of the number of days worked. You can realise from this that the direct ratio of one is equal to the inverse ratio of the other. Hence you can write,

ratio of the number of days worked = inverse ratio of the number of workers,

or, inverse ratio of the number of days worked = ratio of the number of workers.

Let the required number of days worked be x. Then the proportion will be

$$8 : x = 20 : 15 \text{ ..... (i)}$$

or, $x : 8 = 15 : 20$ .... (ii)

Solving any of them, we will get

$$\frac{8}{x} = \frac{20}{15}$$

or, $20x = 8 \times 15$

or, $x = \frac{8 \times 15}{20} = 6$

i.e., 20 workers will make an instrument in 6 days.

You have seen the principle of solving a problem by method of the rule of three with the help of proportionality. Now you observe a brief and easy application of the rule of three.

Example 3. If 4 frocks can be made from 12 metres of cloth, then how many frocks can be made from 18 metres of cloth ?

The problem may be posed as

Quantity of cloth (in metres)	No. of frocks
12	4
18	Required value (x)

Example 4. A hostel with 45 students had a stock of rice for 30 days. After 6 days 9 more students were accommodated in the hostel. For how many days will the remaining stock of rice provide them ?

After 6 days there will be provision for rice for (30–6) days i.e. for 24 days of 45 students.

So, the problem may be pased as

No. of Students	No. of days
45	24
(45+9) = 54	Required value (x)

The analysis of these two examples will reveal the following features :

(1) In these two examples (Examples 3 and 4), there are two things related to each other. In example 3, the things are quantity of cloth and number of frocks and in example 4, the things are number of students and number of days.

(2) Further observe that in both the examples two values of one thing are given, while one value of the other thing is given and another value is to be found out. For example, in example 3, the values of quantity of cloth are given in two cases which are 12 metres and 18 metres, while in case of number of frocks 4 is the value in the first case and the value in the second is to be found out. Similarly, in example 4, the number of students in two

cases are respectively 45 and 54. But the number of days is 24 in the first case and that is to be determined in the second case.

Now let us solve these two problems with the help of proportion. In example 3, if the quantity of cloth increases, the number of frocks will increase. So the proportion will be

$4 : x = 12 : 18$

or, $\frac{4}{x} = \frac{12}{18}$

of, $12x = 4 \times 18$

$\therefore x = \frac{4 \times 18}{12}$

or, $\boldsymbol{x = 4 \times \frac{18}{12}}$

in example 4, with the increase of number of students the number of days will decrease. So the proportion will be

$24 : x = 54 : 45$

or, $\frac{24}{x} = \frac{54}{45}$

of, $54x = 24 \times 45$

$\therefore x = \frac{24 \times 45}{54}$

or, $\boldsymbol{x = 24 \times \frac{45}{54}}$

Observe the last steps of these two solutions, which are printed in bold types

In example 3,	In example 4,
$x = 4 \times \frac{18}{12}$	$x = 24 \times \frac{45}{54}$

It is observed that they have a common feature and that is

Required value = Value in the first case of that thing whose value in the second case is to be determined × a fraction formed by the two values of the other thing.

For example, in example 3,
Required number of frocks

$$= \text{given number of frocks} \times \frac{\text{one value of quantity of cloth}}{\text{other value of the quantity of cloth}}$$

In example 4,
Required number of days

$$= \text{given number of days} \times \frac{\text{one value of the number of workers}}{\text{other value of the number of workers}}$$

The characteristic of the common feature of two solutions is that—in both cases the given value of the thing, whose another value is to be determined, is to be multiplied by a fraction formed by the given values of the other thing. So the formation of this fraction is very much important as the solution depends on it. If this tact is once acquired, a very berief method for rule of three will be obtained.

Now see how you will have to form the fraction :

In example 3, you will be able to identify that required number of frocks will be more than 4 as the quantity of cloth increases from 12 metres to 18 metres. Therefore, 4 has been multiplied by a number greater than 1. This number has been obtained by forming a fraction $\frac{18}{12}$, involving the two given values of the quantity of cloth. Obviously, $\frac{18}{12}$ is greater than 1.

[*Remark :* when denominator is less than the numrator, the fraction is greater than 1.]

In example 4, you will be able to identify that the number of days will be less than 24, as the number of students increases from 45 to 54; therefore, 24 has been multiplied by number less than 1. This number has been obtained by forming a fraction $\frac{45}{54}$, involving the two given values of the number of students. Obviously, $\frac{45}{54}$ is less than 1.

[*Remark* : when denominator is greater than the numerator, the fraction is less than 1.]

— On the basis of this discussion, the solutions of the same problems are shown one after another by forming proportion and by shorter method.

Example 5. A train travels 130 km in 4 hours with a uniform speed. How far will the train travel in 7 hours moving with the same speed ?

The problem may be posed as

Time (in hours)	Distance (in km)
4	130
7	?

Let the required distance be x km.

By method of Proportion : [Here, with the increase of time the distance will increase. So they will be directly proportional.]

$130 : x = 4 : 7.$

or, $\frac{130}{x} = \frac{4}{7}$

of, $4x = 130 \times 7$

$x = \frac{130 \times 7}{4}$

$= 227{\cdot}5$

$\therefore$ The train will travel 227·5 kms in 7 hours.

By shorter method

[The train will travel a distance more than 130 kms. Therefore, 130 is to be multiplied by a fraction $\frac{7}{4}$, greater than 1, formed by 4 and 7.]

$\therefore x = 130 \times \frac{7}{4}$

$= 227{\cdot}5$

$\therefore$ The train will travel 227·5 kms in 7 hours.

Example 6. There was provision for 7 days for 375 villagers confined in a village due to flood. After one day 150 women and children were removed to a safe place. For how many days will the remaining persons be provided with the remaining food ?

After one day there will be provision for (7—1) days, i.e., for 6 days for 375 persons. But because 150 persons were removed after one day the number of remaining persons will be (375—150) = 225.

Hence the problem may posed as :

No. of persons	Provision for no.of days
375	6
225	?

Let the required number of days be x

By method of proportion :

[Here, with the decrease of number of persons, the same amount of food will provide for more days. So, ratio of the number of persons = inverse ratio of the number of days.

$\therefore x : 6 = 375 : 225$

or, $\frac{x}{6} = \frac{375}{225} = 10$ or, $x = \frac{6 \times 375}{225}$

$\therefore$ The remaining food will provide for 10 days

By shorter method :

[As the number of persons has changed from 375 to 225, the number of days will be more than 6. Therefore, 6 is to be multiplied by a fraction $\frac{375}{225}$.

$\therefore x = 6 \times \frac{375}{225} = 10$

$\therefore$ The remaining food will provide for 10 days.

Look at some more examples which are solved only by shorter method.

Example 7. A compositor has composed 196 lines in 7 hours. If he works for 4 hours more as overtime, how many lines will he be able to compose in that time ?

The problem may be posed as,

Time in hours	No. of lines
7	196
4	x

(From the problem it is understood that the value of x will be less than 196, therefore the fraction will be $\frac{4}{7}$)

$$\therefore x = 196 \times \frac{4}{7} = 112$$

$\therefore$ 112 lines will be composed in 4 hours.

Example 8. 50 men take 18 days to dig a pond. If the pond would have to be dug in 15 days, how many men will have to be employed ?

The problem may be posed as

No. of men	No. of days
50	18
x	15

(The value of x will be greater than 50, therefore the fraction will be $\frac{18}{15}$)

$\therefore x = 50 \times \frac{18}{15} = 60$

$\therefore$ 60 men would have to be employed.

Broader application of Rule of Three :

In the above problems, two kinds of quantities which were mutually related were given. But if in any problem, more than two kinds of mutually related quantities are given, how can the problem be solved by applying rule of three ?

Example 9. Rs 132·00 is required to purchase 12 metres of a kind of printed cloth 96 cm broad. How many rupees will be required to purchase 9 metres of same kind of printed cloth 112 cm broad ?

Breadth (cm)	Length (metres)	Price (rupees)
96	12	132
112	9	x

Here it will be taken as the ratio of the values of the things whose one value is to be determined is equal to the compound ratio of the values of given things. Of course, in forming the compound ratio the relation of each of the given things with the thing whose value is to be determined is to be considered separately. This relation may be of two types— (1) direct relation between the two (2) inverse relation between the two.

In case of present example, the breadth and length of the printed cloth is directly related with the price. Hence the proportion will be

$132 : x = (96 \times 12) : (112 \times 9),$

or, $\frac{132}{x} = \frac{96 \times 12}{112 \times 9}$

$$\text{or, } x = \frac{132 \times 112 \times 9}{96 \times 12} = 115^{\cdot}5$$

∴ The required price is Rs. 115·50

By shorter method :

[When considered with respect to breadth, for increase in breadth, price will increase. Therefore, in this case the fraction will be $\frac{112}{96}$. Again when considered with respect to length, for decrease in length, the price will decrease. Therefore, in this case the fraction will be $\frac{9}{12}$.]

∴ $x = 132 \times \frac{112}{96} \times \frac{9}{12} = 115{\cdot}5$ ∴ The required price is Rs. 115·50

Example 10. 16 bighas of land can be cultivated in 8 days by using 6 ploughs. In how many days will the cultivation be complete, if 9 ploughs are used to cultivate 21 bighas of land ?

The problem may be posed as

No. of ploughs	Days	Quantity of land
6	8	16
9	x	21

[Here the number or ploughs is inversly related with the number of days, but quantity of land is directly related with the number of days.]

So the proportion will be

$$8 : x = (9 \times 16) : (6 \times 21)$$

$$\text{or, } \frac{8}{x} = \frac{9 \times 16}{6 \times 21}$$

$$\text{of, } x = \frac{8 \times 6 \times 21}{16 \times 9} = 7$$

∴ the required no. of days = 7

[With the increase in the number of ploughs, the number of days will decrease. Hence the fraction of days will be $\frac{6}{9}$; with the increase in the quantity of land, the number of days will increase. Hence the fraction will be $\frac{21}{16}$.]

$$\therefore x = 8 \times \frac{6}{9} \times \frac{21}{16} = 7$$

∴ the required no. of days = 7

Example 11. 25 workers can make an instrument in 12 days working 8 hours a day. If 20 workers are employed and they work 6 hours a day, then how many days will be required to make such an instrument ?

The problem may be posed as,

Hours	Days	Workers
8	12	25
6	x	20

[Here both the numbers of hours and workers are inversely related with number of days.]

Hence the proportion will be

$$12 : x = (6 \times 20) : (8 \times 25)$$

$$\text{or, } \frac{12}{x} = \frac{6 \times 20}{8 \times 25}$$

$$\text{or, } x = \frac{12 \times 8 \times 25}{6 \times 20} = 20$$

∴ the required number of days = 20

By shorter method :

[If the number of hours decreases, the number of days will increase, so the fraction will be $\frac{8}{6}$. Again if the number of workers decreases, the number of days will increase, so the fraction will be $\frac{25}{20}$.]

$$\therefore x = 12 \times \frac{8}{6} \times \frac{25}{20} = 20$$

∴ the required no, of days = 20

Now see one example worked out only in shorter method.

Example 12. If water is poured in a tank, 5 metres long and 3 metres broad, by a pump of diameter 6 cm, the water level will be 2·4 metres deep. How much time will be required to completely fill an empty tank, 4 metres long 3·6 metres broad and 2 metres deep by a pump of diameter 5 cm ?

The problem may be posed as

Diametre (cm)	Time (minutes)	Length (metres)	Breadth (metres)	Depth (metres)
6	25	5	3	2·4
5	x	4	3·6	2

$$\therefore x = 25 \times \frac{6}{5} \times \frac{4}{5} \times \frac{3{\cdot}6}{3} \times \frac{2}{2{\cdot}4} = 24$$

∴ the required time = 24 minutes

Exercise 3

1. If the cost of 12 kg of pulses is Rs. 48, what is the cost of 30 kg of pulses ?

2. 12 men can do a piece of work in 15 days. How long will 10 men take to do the same work ?

3. A tailor makes 9 frocks out of 12 metres of printed cloth. For 150 frocks how much cloth would he buy ?

4. A quantity of food provides 24 men for 20 days. How long will it last if the number of men is 40 ?

5. A factory produces 14,600 bulbs in 365 days. How many bulbs will be produced in the factory in 140 days ?

6. 840 gm of chromium is required to make 7 kg of stainless steel. How much chromium is required to make 11 kg of stainless steel ?

7. Shibani goes from Bali-crossing to Chinsura in 35 minutes and the distance is 2·5 km. How long will she take to reach Chandannagar, 10 km away, if she goes at the same speed ?

8. In a camp of 4,000 men there was provision for food for 190 days. After 30 days 800 men went away to an other place. How many days will the rest of the food provide the remaining men ?

9. In a co-operative farm 42 peasant-members can plough the total land in 24 days. But during the planting season 6 of them fell ill. How many days was required to plough the total land ?

10. Rs. 9·50 is required to travel 250 km by rail. How far can you travel for Rs. 57 ?

11. There are 16 machines in a factory and it can manufacture 1,000 machine-parts n 27 days. Two more similar machines are installed in the factory. How many days will now be required to manufacture the same number of machine-parts in that factory ?

12. 368 bundles of hay are required to make the two slanting roofs of a shed, 4 meters long. If the length of roofs are increased by 50 cm, how many more bundles of hay will be required ?

13. It was decided at the Panchyet to clear-up 3 silted ponds in the field to improve the irrigation factility. It was also worked out that 25 labourers are necessary to complete the job in 36 days. But apprehending early rain the work was proposed to be completed in 10 days. How many labourers are to be engaged to complete the work by that time ?

14. 4 Kahans of hay is required to feed 8 cows for 15 days. According to this calculation how many Kahans of hay is to be stocked to feed 10 cows for 72 days during the rainy season ?

15. The transportation cost of carrying 33 quintals of goods to a destination 150 km away is Rs. 42. What is the weight of the goods for which the cost of carrying through a distance of 120 km is Rs. 35 ?

16. 18 bighas of land may be irrigated in 8 days by a shallow tube-well pump working 15 hours a day. How many days will it take to irrigate 21 bighas of land by operating the pump 10 hours a day ?

17. 15 labourers can plough 18 bighas of land in 5 days. If 10 labourers work for 10 days, then how much land will be ploughed ?

18. 5 mason can build a wall of 128 sq. metres in 4 days. How long will it take to build a wall of 320 sq. metres by 10 mason ?

19. A piece of plank of length 4 metres and breadth 3 decimetres costs Rs. 62. Calculate the length of a plank of breadth 4 decimetres having the same thickness, which costs Rs. 186.

20. A cargo ship takes 25 days to go from Calcutta to Cochin and it starts from Calcutta with food for its 36 crews at the rate of 850 gms a day per head. After 13 days they rescued 15 passengers from a sinking ship and increased the speed of the ship so that it could reach Cochin within next 10 days. How much food should they now consume per day per head so that it is just enough till they reach Cochin ?

21. In a project of food starts building a village road under the for work a team of 250 workers leadership of the village-panchayat. After 12 days it was found that only $\frac{2}{7}$th part of the work was completed.To complete the rest in 25 days how many more men should be employed ?

22. In a hand-written manuscript of 105 pages there are 25 lines per page containing 8 words per. line. If the book is printed in such a manner that there are 30 lines per page and each line contains 10 words, calculate the total number of pages the book will have.

23. A laboratory has to pay Rs. 22·50 as gas-bill for 30 days if it uses 35 gas-burners for 6 hours a day. Find the amount to be paid if it uses 30 gas-burners for 18 days using them 7 hours a day.

24. In a handloom weavers' co-operative, 14 weaver-members can process 210 saries in 12 days. In the puja season they got an order to supply 300 saries within 10 days. How many extra weavers should they employ to help them to carry out the order in time ?

25. Some workers working 6 hours a day can dig a canal of length 70 m, breadth 3 m and depth 2 m in 10 days. Calculate how long will it take to dig a canal of length 140 m, breadth 4 m and depth 2·5 m, if they work 8 hours a day.

26. A power loom produces $2\frac{1}{4}$ times more than a handloom. If 12 handloom can produce 1,080 metres of cloth in 18 days, then how many powerlooms should be operated to produce 2,700 metres of cloth in 15 days ?

27. A 5 H.P. pump can irrigate 36,000 litres of water in 8 hours. How long should a 7 H.P. pump be operated to irrigate 63,000 litres of water ?

28. A farmers' co-oprative has bought a tractor. Earlier it took 36 days to till 2,400 bighas of land with 25 ploughs. Now one tractor can till half of the land in 30 days. Calculate how many ploughs are equivalent to a tractor.

Division into proportional parts :

Let us suppose that a mason mixes 10 kg of cement with 55 kg of sand to prepare mortar for plastering a wall.

Thus, in (10 + 55) kg i.e., 65 kg of the mortar, there is 10 kg of cement and 55 kg of sand.

Now to express the amount of cement and sand as the fraction of the total quantity, we write

Part of cement in the mixture $= \frac{10}{10+55} = \frac{10}{65} = \frac{2}{13}$

And, part of sand in the mixture $= \frac{55}{10+55} = \frac{55}{65} = \frac{11}{13}$

Now let us proceed to find the quantity of cement and sand in 26 kg of the mixture.

Since the mixture is homogeneous, in any portion of it the ratio of the quantities of cement and sand will be the same.

We know that in the orignal mixture,

cement : sand = 10 : 55 = 2 : 11

So we can say,

Part of cement $= \frac{2}{2+11} = \frac{2}{13}$

Part of sand $= \frac{11}{2+11} = \frac{11}{13}$

$\therefore$ each part of the mixture contains $\frac{2}{13}$ part of cement and $\frac{11}{13}$ part of the sand.

$\therefore$ if we take 26 kg of the mixture,

the quantity of cement $= \frac{2}{13}$ of 26 kg. $= \left(26 \times \frac{2}{13}\right)$ kg = 4 kg. and the quantity of sand $= \frac{11}{13}$ of 26 kg. $= \left(26 \times \frac{11}{13}\right)$ kg. = 22 kg.

In any mixture, the part of each individual ingredient present in one part of the mixture, obtained from the given ratio of the ingredients in the mixture, is known as proportional part of the ingredient.

In the above example, $\frac{2}{13}$ and $\frac{11}{13}$ are proportional parts of cement and sand respectively.

Consider the following examples illustrating the above fact :

Example 1. A man spends a part of his monthly income and saves a part of it. The ratio of his expenditures to the savings is 27 : 2 and his monthly income is Rs. 725. What are his monthly expenditures and savings ?

It is given that, expenditures : savings = 27 : 2

$\therefore$ proportional part of expenditures $= \frac{27}{29}$

and proportional part of savings $= \frac{2}{29}$

$\therefore$ when the monthly income is Rs. 725,

his expenditures $= \frac{27}{29} \times$ Rs. 725 = Rs. 675

and his savings $= \frac{2}{19} \times$ Rs. 725 = Rs. 50

Example 2. The total population in a village is 1,428 and the ratio of women to men is 25 : 26. Find the number of men and the number of women in that village.

Here the ratio of women : men = 25 : 26

$\therefore$ Proportional part of women $= \frac{25}{51}$ and proportional part of men $\frac{26}{51}$

$\therefore$ no. of women $= \frac{25}{51} \times 1{,}428 = 700$

and no. of men $= \frac{26}{51} \times 1{,}428 = 728$

So long we have discussed ratio of two quantities. Now we will consider the ratio of more than two quantities and will see how to find the proportional parts thereof.

Example 3. A man donated Rs. 2,000 from his provident fund savings for the improvement of the village school. The remaining portion is divided among his wife, son and daughter in the ratio 5 : 4 : 4. If his total savings be Rs. 11,750, find the money received by each.

The money left after the donation

Rs. (11,750 – 2,000) = Rs. 9,750

It is given that,

Wife's share : son's share : daughter's share = 5 : 4 : 4

Proportional parts will be,

$$\text{Wife} = \frac{5}{5+4+4} = \frac{5}{13} \text{ part.}$$

$$\text{Son} = \frac{4}{5+4+4} = \frac{4}{13} \text{ part.}$$

$$\text{Daughter} = \frac{4}{5+4+4} = \frac{4}{13} \text{ part.}$$

∴ out of Rs. 9750,

$$\text{money received by wife} = \frac{5}{13} \times \text{Rs. } 9{,}750 = \text{Rs. } 3{,}750$$

$$\text{money received by son} = \frac{4}{13} \times \text{Rs. } 9{,}750 = \text{Rs. } 3{,}000$$

$$\text{money received by daughter} = \frac{4}{13} \times \text{Rs. } 9{,}750 = \text{Rs. } 3{,}000$$

Example 4. Divide Rs. 81 among three friends in such a way that the second friend gets twice that of the first and the third friend gets half of the sum received by the other two.

According to the given condition,

If first friend gets 1, second friend gets 2

and third friend gets $\frac{1+2}{2} = \frac{3}{2}$.

∴ the ratio of their shares is

$$\text{1st : 2nd : 3rd} = 1 : 2 : \frac{3}{2} = 2 : 4 : 3$$

∴ the money received by them will be.

1st friend gets $\frac{2}{9} \times$ Rs. 81 = Rs. 18

2nd friend gets $\frac{4}{9} \times$ Rs. 81 = Rs. 36

3rd friend gets $\frac{4}{9} \times$ Rs. 81 = Rs. 27

In our daily life we often find fluctuations (either decrease or increase) in items which are made of two or more correlated constituents. In such cases it is required to find out the fluctuations (decrease of increase) of the quantity of the constituents when there is a change in the quantity of the given item.

We can easily solve such problems with the help of proportional parts.

Example 5 . Two friends Sunil and Paresh are staying in a two-roomed flat, sharing the rent of Rs. 250 per month. Sunil pays Rs. 130 for using the bigger room and Paresh pays Rs. 120 for using the smaller one. If the rent increases by Rs. 50 from next month, how will Sunil and Paresh share the new rent of Rs. 300 among themselves according to previous proportional parts ?

In this case, due to the increase in the rent both the friends will have to pay more money than before and the amount to be paid by each friend will be calculated according to the same proportional parts as before.

As we know they used to share the rent in the ratio

Sunil : Paresh = 130 : 120 = 13 : 12,

Accordingly, when the rent is Rs. 300

Sunil will pay $\left(\frac{13}{25} \times \text{Rs. } 300\right)$ = Rs. 156

And Paresh will pay $\left(\frac{12}{25} \times \text{Rs. } 300\right)$ = Rs. 144.

[*Remark* : You can solve the problems on proportional parts algebraically as in case of problems on ratio. You will have to use after determining the ratios. Algebraic methods may also be used in a general way.]

Exercise 4

1. The ratio of literate to illiterate people of the village Manikchak is 1 : 4. If the population of the village be 2,655; what is the number of literates and illiterates in that village ?

2. For cultivation of paddy organic and chemical fertilizers are used in the ratio 9 : 2; if 22 bags of mixed fertilizer is required per bigha of land, then how much of each type of fertilizer is required for 8 bighas of land?

3. Iron and carbon are in the ratio of 49 : 1 in a particular type of steel. How many quintals of carbon are there in 250 quintals of that type of steel.

4. Divide Rs. 720 was divided among Ram, Shyam and Jadu in such a way that the ratio of their shares be 3 : 7 : 14. How much did each receive ?

5. Divide Rs. 940 among Biltu, Shiltu and Aloke in such a way that their shares are in the ratio $1\frac{1}{2} : 3 : 3\frac{1}{2}$. How much did each receive ?

6. A Panchayet proposed to divide the government grant of Rs. 84,800 in the ratio of 2·5 : 4 : 1·5 on three main heads. What will be the amount of expenditure for each separate head ?

7. In pure water the ratio of the weights of Oxygen to that of Hydrogen is 88·9 : 11·1. What are the quantities of Oxygen and Hydrogen in 2 quintals of water ?

8. In a certain year the ratio of the literate to the illiterate Indians was 33 : 67. What were the number of literates and illiterates, if the total population was 62 crores in that year ?

9. Out of 3,650 villagers in a village, the ratio of the number of adults to the number of minors is 7 : 3. Find the number of adults in the village.

10. The monthly salary of a man is Rs. 2,150. Out of which he pays Rs. 550 as repayment of loan and the rest is divided in the ratio 3:1 for family expenses and educational expenses of his sons and daughters. Find how much he spends on each item.

11. A consumers' co-operative gives Rs. 42,150 as dividend to the members and the rest is given as remuneration to the employees and loan to the public in the ratio 7 : 18. If in a certain year the co-operative is in a position to spend Rs. 77,575; find the amount of loan it can give.

12. Rs. 700 is divided amongst A, B and C in such a way that the ratio of the amounts of A and B is 2 : 3 and that of B and C is 4 : 5. How much does each of them receive ?

13. After deduction of wealth-tax of Rs. 250 from Rs. 25,750; two brother receive the money in the ratio 3 : 2 and each of them gives to their sister Rs. 2,000. What is the ratio of the money received by the two brothers and their sister ?

14. In a certain year Rammohan Library received as subscription Rs. 3,350, government aid 75,350 and Rs. 1,300, by selling old newspapers etc. If the expenses for buying books, binding books and salary to the employees are in the ratio 15 : 3 : 2, find the amount of expenses for each item.

15. Sabita De gets a monthly salary of Rs. 1,950. Out of which she deposits Rs. 150 in the provident fund account and spends the rest for family expenses and house rent in the ratio 5 : 1. Her salary increases to Rs. 2,550; but the provident fund contribution and house rent remain the same and she spends the rest for family expenses. Find the present ratio of her family expenses to house rent.

16. Out of 555 inhabitants of Maheshpur 111 are literate and at Dihibayra 390 out of 450 are illiterate. What is the ratio of illiterate to total population in each village and in the two villages taken together ?

17. In a co-operative lathe-machine workshop the yearly net profit is divided in the ratio 5 : 3 as dividend to the member-workers and to the fixed deposit account. If in a particular year the net profit be Rs. 20,000; what amount will go to the fixed deposit account ?

18. Of 132 examinees of a certain school the ratio of successful to unsuccessful students is 9 : 2. If 4 more students passed what would have been the ratio of successful to unsuccessful students ?

19. Last year a peasant spent Rs. 1,400 for cultivation of paddy and at that time the ratio of his expenses and income from cultivation was 2 : 5. This year by cultivating high yielding variety of paddy his expenses and income have increased by $1\frac{1}{2}$ and 2 times respectively. What is the net increase in his income this year ?

20. In a certain assembly constituency the total number of voters is 45,000 and on the polling day the following data are collected :

(a) votes polled : absentees = 49 : 1

(b) no vote is cancelled.

(c) votes polled by the winner : votes poled by the looser = 16 : 9,

(i) What is the margin of votes by which the winning candidate won the election ?

(ii) Total votes polled : votes polled by the winner = ?

21. An agricultural farm allots its 35 bighas of land for growing paddy and jute in the ratio 4 : 3. If the profit from paddy and jute be Rs. 2,500 and Rs. 2,800 per bigha respectively, obtain the ratio of total profit from paddy to total profit from jute.

22. The actual expenditures for the developent of Bidhan Nagar for four consecutive years were in the ratio 2 : 3 : 4 : 2. If the total expenditure for those four years is Rs. 121 crores, then what was the expenditure for the 3rd year ? How much less or more was the total expenditure for the 1st year and the 2nd year in comparison to that of the 3rd year ?

Partnership Business :

Partnership business is one of the common forms in the business-field, where two or more persons run a business jointly. The main principles of partnership business are as follows:

(1) The partners mutually agree upon the principle of sharing the capital.

(2) The partners also mutually settle the profit sharing ratio among themselves. If there is no profit sharing ratio in the agreement, the profit will be shared by the partners according to the ratio of the capital contributed by them. As for example, if the capital of two partners are Rs. 2,000 and Rs. 3,000 respectively and if the profit sharing ratio is not mentioned in the agreement, the profit will be distributed among the two partners in the ratio 2 : 3 i.e. in the ratio of the capital contributed by them.

(3) If all the partners or any one of them give any extra labour in the management of the business, then the remunerations for those services are sometimes pre-agreed.

In the previous chapter you have learnt to solve problems with the help of proportional parts. In the following examples, you will see how the same principle of proportional parts may be applied to solve the problems of profit sharing in a partnership business.

Example 1. A, B and C started a partnership business agreeing to share the profit according to the ratio of the capital contributed by them. The firm made a profit of Rs. 1,155. If the capital invested by A, B and C are Rs. 5,000; Rs. 4,500 and Rs. 7,000 respectively, what will be their individual share in the profit ?

The ratio of their capital is :

A : B : C = 5000 : 4500 : 7000 = 10 : 9 : 14

∴ Shares of the profit will be as follows :

A receives $\left(\frac{10}{33} \times \text{Rs. } 1155\right) = \text{Rs. } 350$

B " $\left(\frac{9}{33} \times \text{Rs. } 1155\right) = \text{Rs. } 315$

C " $\left(\frac{14}{33} \times \text{Rs. } 1155\right) = \text{Rs. } 490$

Example 2. Three friends start a business with capitals of Rs. 3,000; Rs. 3,500 and Rs. 2,500 respectively. They made an agreement that $\frac{1}{3}$rd of the net profit will be shared equally among themselves according to ratio of their capitals. At the end of the year, if the profit incurred is Rs. 810, then (i) how much will each receive ? (ii) What will be the ratio of the amounts received by them ?

According to the conditions of the agreement $\frac{1}{3}$rd of the net profit i.e. $\frac{1}{3}$ rd of Rs. 810 = Rs. 270 will be equally divided among the three friends.

∴ Each will receive Rs. 270 ÷ 3 = Rs. 90 out of it.

Remaining portion of the profit = (Rs. 810 – Rs. 270)
= Rs. 540.

This profit will be divided among the three friends according to the ratio of their capitals.

The ratio of their capitals is,

1st : 2nd : 3rd = 3000 : 3500 : 2500 = 6 : 7 : 5

∴ 1st friend will receive $\left(\frac{6}{18} \times \text{Rs. } 540\right) = \text{Rs. } 180$

2nd friend will receive $\left(\frac{7}{18} \times \text{Rs. } 540\right) = \text{Rs. } 210$

3rd friend will receive $\left(\frac{5}{18} \times \text{Rs. } 540\right) = \text{Rs. } 150$

∴ out of the total profit their receipt will be,

1st friend = Rs. (180 + 90) = Rs. 270

2nd friend = Rs. (210 + 90) = Rs. 300

3rd friend = Rs. (150 + 90) = Rs. 240

∴ the ratio of their shares of profit will be

1st : 2nd : 3rd = 270 : 300 : 240

9 : 10 : 8

Example 3. Four friends start a business with Rs. 7,000; Rs, 8,500; Rs. 10,000 and Rs. 15,000 respectively as capital. They decided that the first three would carry out the responsibility of managing the business for 5 months, 4 months and 3 months respectively. They further decided that $\frac{1}{4}$th of the total profit would be shared in proportion of their services and the rest in the ratio of their capital. If at the end of 1 year the profit is Rs. 3,888; find the money received by each from the profit.

According to the conditions,

$\frac{1}{4}$th of the profit i.e., $\frac{1}{4}$th of Rs.3888 = Rs. 972 will be divided according to the services rendered by them.

The 4th one did not take part in the management.

∴ this part will be shared by the first three friends.

The ratio of their services is,

1st : 2nd : 3rd = 5 : 4 : 3

∴ 1st friend will receive $\left(\text{Rs. } 972 \times \frac{5}{12}\right)$ = Rs. 405

2nd friend will receive $\left(\text{Rs. } 972 \times \frac{4}{12}\right)$ = Rs. 324

3rd friend will receive $\left(\text{Rs. } 972 \times \frac{3}{12}\right)$ = Rs. 243

Now the rest of the profit i.e., Rs. (3888 – 972) = Rs. 2916 will be shared according to ratio of their capitals.

The ratio of their capitals is

1st : 2nd : 3rd : 4th = 7000 : 8500 : 10,000 : 15, 000
= 14 : 17 : 20 : 30

∴ 1st friend will receive $\left(\text{Rs. } 2916 \times \frac{14}{81}\right)$
= Rs. 504

2nd friend will receive $\left(\text{Rs. } 2916 \times \frac{17}{81}\right)$
= Rs. 612

3rd friend will receive $\left(\text{Rs. } 2916 \times \frac{20}{81}\right)$
= Rs. 720

4th friend will receive $\left(\text{Rs. } 2916 \times \frac{30}{81}\right)$
= Rs. 1080

∴ out of the total profit their receipt will be,

1st friend = Rs. (405 + 504) = Rs. 909
2nd friend = Rs.(324 + 612) = Rs. 936
3rd friend = Rs. (243 + 720) = Rs. 963
4th firend = Rs. 1080

Example 4. In a certain year on 1st January A, B and C started partnership business contributing Rs. 5,000; Rs. 6,000 and Rs. 7,000 respectively as capital. On 1st of April *B* contributed Rs. 1,000 more, but on 1st of June *C* withdrew Rs. 1,000 from the capital. The net profit till 31st December was Rs. 3,924. How much did each receive according to ratio of their capitals ?

In the above example, it is clear that the capital of all the partners did not roll in the business equally throughout the whole year. Therefore, to find the ratio of the capital of different partners the time factor should also be considered along with the amounts of capital.

To receive same amount of profit in one month as was recevied by investment of A's capital of Rs. 5,000 in 12 months, capital would have been required Rs. (5000 × 12) = Rs, 60,000

Similarly, B's Rs. 6000 and Rs. 7000 rolled for 3 months and 9 months respectively.

∴ B's capital in respect of 1 month

= Rs. (6000 × 3) + Rs. (7000 × 9) = Rs. 81000.

Again, C's Rs. 7000 and Rs. 6000 rolled for 5 months and 7 months respectively.

∴ C's capital in respect of 1 month

= Rs. (7000 × 5) + Rs. (6000 × 7) = Rs. 77000

∴ the ratio of their capitals will be,

A : B : C = 60000 : 81000 : 77000

= 60 : 81 : 77

∴ the profits received by each are as follows :

A earned $\left(\text{Rs. } 3924 \times \frac{69}{218}\right) = \text{Rs. } 1080$

B earned $\left(\text{Rs. } 3924 \times \frac{81}{218}\right) = \text{Rs. } 1485$

C earned $\left(\text{Rs. } 3924 \times \frac{77}{218}\right) = \text{Rs. } 1386$

[Remark : In case of partnership business also you could have used algebra. since problems are being solved after finding ratio.]

Exercise 5

1. Two friends, Netai and Jabbar, started a partnership business investing Rs. 1,500 and Rs. 2,500 respectively. The profit at the end of the year was Rs. 1,680. How much profit did each partner earn ?

2. Abed, Uttam and Jafer started a partnership business investing Rs. 5,000, Rs. 6,000, Rs. 5,500 as capital respectively. If the profit at the end of the year is Rs. 4,950, find the profit earned by each of them.

3. A, B and C started a business with Rs. 1,500, Rs. 1,000 and Rs. 1,750 respectively as capital. At end of the year there was a loss of Rs. 425. How much did each lose ?

4. Jadu and Madhu purchased a car for Rs. 25,000 and sold the same for Rs. 26,250. If at the time of purchase Jadu paid $1\frac{1}{2}$ times as much as Madhu, how much did Madhu receive out of the profit ?

5. Priyabrata, Priyatosh and Prititosh started a partnership business investing Rs. 500, Rs. 600 and Rs. 700 respectively as capital. At the end of one year they incurred a loss of Rs. 180. They decided to pay for the loss to keep the capital intact. How much should Priyatosh pay ?

6. Three workers Sukanta, Sushanta and Subir started a lathe machine shop with capital of Rs. 6,000, Rs. 9,100 and Rs. 5,200 respectively. At the end of one year the profit was Rs. 14,400. If $\frac{2}{3}$ of the profit is divided equally and the rest according to ratio of their capitals, then find the profit earned by each of them.

7. Three friends Tapas, Nihar and Biswajyoti started a factory investing Rs. 4,000, Rs. 5,000, Rs. 6,000 respectively and the rest of the capital was taken from a bank as loan. At the end of the year, the profit was Rs. 6,700 out of which they paid the bank instalment of Rs. 2,500 and the rest of the profit was shared by the partners according to their capitals,. How much did each receive out of the profit ?

8. Keshab, Madhab and Jadab started a partnership business with Rs. 15,000; Rs, 18,000 and Rs. 17,000 respectively. Due to shortage of fund they borrowed some money from the Industrial Development Corporation for which they had to pay the Industrial Development Corporation an instalment of Rs. 15,000 every year. At the end of one year, if the profit be Rs. 76,500 how much would each receive after paying the instalment of Industrial Development Corporation ?

9. Three friends took loans of Rs. 1,500; Rs. 2,000 and Rs. 1,250 from a co-operative bank on the condition that if the loan is paid back within two years, no interest will be charged and by the loan money they bought two rickshaws. After two years they had a net income of Rs. 1,600. They divided the income in the ratio of their capitals and each repaid his respective bank loan. Find the amount each will have now in their hand and the ratio of their amounts.

10. Three friends purchased a bus investing Rs. 60,000; Rs. 75,000 and Rs 33,000 respectively. Worked as the first driver and the other two as conductors of the bus. They decided that $\frac{3}{5}$ th of the total income should be divided in the ratio 3 : 2 : 2 for their work and

the rest in ratio of their capitals. If the income of a certain month be Rs. 14,630 of profit would each share.

11. At the beginning of the year A and B started a business investing Rs. 12,000 and Rs. 15,000 respectively. After 5 months A invested Rs. 2,000 more. At the end of the year the profit was Rs. 13,858. How much of profit will each share ?

12. In a co-operative lathe machine workshop, out of five member-workers three invested Rs. 2,000 each and two Rs. 3,000 each. They decided to share $\frac{8}{9}$ th of the annual profit equally and the rest in ratio of their capitals. Find the share of profit of each, if the profit at the end of the year be Rs. 8,370.

13. On 1st January Mahendra, Mannan and Rupchand started a grocery shop contributing Rs. 7,000, Rs. 6,000 and Rs. 8,000 respectively. On 1st April Mannan contributed Rs. 1,000 more and on 1st July Mahendra Rs. 5000 more. But on 1st July Rupchand withdrew Rs. 500. If the net profit upto 31st December be Rs. 9,744, calculate the share of profit each received.

14. At the beginning of the year Gobinda and Mukunda purchased a minibus with capital of Rs. 60,000 and Rs. 75,000 respectively. After four months when their friend Ananda contributed Rs, 40,500, Govinda and Mukunda withdrew that sum in their capital ratio. If the profit at the end of the year be Rs. 19,575; what is the share of profit of each ?

15. Three artisans of Kumartuli took a loan of Rs. 10,000 and started a workshop for clay modelling. They decided that each after paying the bank instalment of Rs. 2810, half of the profit should be divided according to the number of days each worked and the other half should be shared equally. In the last year they worked for 300, 275 and 350 days respectively and the net profit was Rs. 13,910. Calculate the share of profit of each.

16. Amal, Anup and Anwar decided to start a business with a capital of Rs. 75,000. They collected part of the capital in the ratio of 3 : 4 : 2 and still Rs. 30,000 more was needed. This amount was taken as a loan from a bank on the condition to repay the loan in yearly instalments of Rs. 3,375. They agreed to divide pay the bank instalment in the ratio 3 : 2 : 4 from their respective share of profit,. In the last year there was a net profit of Rs. 39,375. Calculate the amount left to each of them after they have paid their bank instalments.

Mixture (Alligation) :

You all know that in our practical life we sometimes need to mix two or more ingredients (components) to form a mixture. In a particular mixture the proportion in which the different ingredients are mixed is known as the ratio of the ingredients in the mixture. At times it may so happen that we are required to increase or decrease the quanties of those ingredients and as such some problems arise.Let us see how these problems can be dealt with by the help of Ratio and Proportion.

Example 1 : In a particular type of mixed fertilizer the ratio of urea and potash is 2 : 5. In 14 kg of such a fertilizer, if 2 kg of urea is added, what will be the ratio of urea to potash in the new mixture ?

In the 1st mixture, urea : potash = 2 : 5

$\therefore$ in 14 kg of this of type mixed fertilizer,

Quantity of urea is $\left(\frac{2}{7}\times 14\right)$ kg = 4 kg.

Quantity of potash is $\left(\frac{5}{7}\times 14\right)$ kg = 10 kg.

Now if 2 kg of urea is added to the above mixture, the new mixture will contain the following quantities of urea and potash.

Urea = (4 + 2) kg = 6 kg

Potash = 10 kg

$\therefore$ the ratio of the two components in the new mixture will be

urea : potash = 6 : 10 = 3 : 5

Example 2 : In 10 litres of dilute sulphuric acid, the ratio of water and acid is 1 : 4. How much acid is to be added to this solution to make the ratio of water and acid as 1 : 5 ?

Since, Water : acid = 1 : 4

∴ in 10 litres of the mixture the volume of water

$$= \left(\frac{1}{5} \times 10\right) \text{ litres} = 2 \text{ litres}$$

and the volume of acid $= \left(\frac{4}{5} \times 10\right)$ litres = 8 litres.

Let x be the volume of acid to be added,

∴ from the given condition, we have

$$\frac{2}{8+x} = \frac{1}{5}$$

or, $8 + x = 10$

or, $x = 18 - 8$

$\therefore x = 2$

∴ quantity of acid to be added is 2 litres.

Example 3. In two types of powdered detergent the ratio of soda and soap-dust is 2 : 19 and 1 : 11 respectively. If 7 kg of the first type is mixed with 4 kg of the second type, find the ratio of soda to soap-dust in the new detergent mixture.

In the first type of detergent, soda : soap-dust = 2 : 19

∴ in 7 kg, soda contains $\frac{2}{21} \times 7$ kg $= \frac{2}{3}$ kg.

and soap-dust contains $\frac{19}{21} \times 7$ kg $= \frac{19}{3}$ kg

In the second type of detergent, soda : soap-dust = 1 : 11

∴ in 4 kg, soda and $\frac{1}{12} \times 4$ kg $= \frac{1}{3}$ kg

and soap-dust

$= \frac{11}{12} \times 4$ kg

∴ in the new detergent,

$\left(\frac{2}{3} + \frac{1}{3}\right)$ soda = 1 kg

and soap-dust $= \left(\frac{19}{3} + \frac{11}{3}\right)$ kg = 10 kg

therefore, the new ratio will be soda : soap-dust = 1 : 10

Example 4 : In two blends of mixed tea the ratios of Darjeeling and Assam tea are 2 : 7 and 1 : 5 respectively. In what ratio should these two blends of tea be mixed so that the ratio of Darjeeling and Assam tea in the new blend is 1: 4 ?

Let $9x$ kg of the 1st blend is to be mixed with $6y$ kg of the 2nd blend.

[Try to understand why such assumptions are made.]

$9x$ kg of the 1st blend contains $2x$ kg of Darjeeling tea, and 7x kg of Assam tea.

$6y$ kg of the 2nd blend contains 7 kg of Darjeeling tea, and $5y$ kg of Assam tea.

$\therefore$ the new blend will contain ($2x + y$) kg of Darjeeling tea, and ($7x + 5y$) kg of Assam tea.

According to the given condition,

$$\frac{2x + y}{7x + 5y} = \frac{1}{4} \text{ or, } 8x + 4y = 7x + 5y$$

$$\text{Or, } 8x - 7x = 5y - 4y \text{ or, } x = y$$

$$\therefore \frac{9x}{6y} = \frac{9y}{6y} = \frac{3}{2}$$

$\therefore$ 1st blend : 2nd blend = 3 : 2

Exercise 6

1. In a mixed fertilizer the ratio of urea to potash is 3 : 7. In 20 kg of this fertilizer, if 4 kg of uera be added, calculate the new ratio of urea to potash.

2. In an alloy of brass the ratio of copper to Zinc is 5 : 3. In 24 kg of such a quality of brass if 3 kg of copper be added, find the new ratio of copper to zinc.

3. 49 kg of blended tea contain Assam and Darjeeling tea in the ratio 5 : 2. What quantity of Darjeeling tea is to be added to the mixture to make the ratio of Assam to Darjeeling tea 2 : 1 ?

4. In 63 litres of dettol water the ratio of water to dettol is 20 : 1, how much dettol should be added in it to make the ratio 12 : 1 ?

5. A family buys rice and wheat in the ratio 4 : 3 and their total cereal consumption is 49 kg per month. But due to rise in the price of rice it reduces purchase of the quantity of rice and increases that of wheat by the same quantity. Due to this change the ratio of rice to wheat that are bought becomes 1 : 1. What is the reduction in the quantity of rice ?

6. Type material of a printing press are made of an alloy of lead, antimony and tin in the ratio 17 : 3 : 1 what are the amounts of different metals in 3·15 quintals of this alloy ?

7. A mixed fertilizer for paddy field contains ammonia, phosphate and potash in the ratio 11 : 5 : 2. How much quantity of each variety should a peasant buy who uses 16 kg of the mixed fertilizer per bigha for his 9 bighas of land ?

8. 12 metres of cloth was used to make one set of jersy with red and blue coloured cloth. If the ratio of the length of red cloth to that of the blue one is 3 : 2, calcutate the length of red coloured cloth used. If the ratio is altered to 2 : 1, calculate the length of cloth of different colours that will be required.

9. Out of 504 students of a school the ratio of the number of boys to that of girls is 13 : 11; if 12 more girls get admitted what will be the new ratio ?

10. A beverage is prepared by adding 2 litres of orange squash to 12 litres of water. What is the ratio of water to squash in the beverage ? If 0·5 litres of squash be added to it, what will be the new-ratio ?

11. In a particular type of soldering material the ratio of lead to tin is 5 : 4; how much tin should be added to 4·5 kg of this material to make the ratio 1 : 1 ?

12. In 45 litres of phenol-water the ratio of phenol to water is 2 : 23; how much water should be added to it to make the ratio 3 : 37 ?

13. A mason made a mixture of sand and cement in the ratio 7 : 1. After completion of brick laying 72 kg of the mixture was in excess. He then added some more cement to it to make the ratio 6 : 1 and used it for plastering work. How much cement did he add ?

14. In a glass of beverage, syrup and water are in the ratio 3 : 1. What fraction of the amount of beverage should be replaced by water to make quantity of syrup and water equal ?

15. In two types of detergent powder the ratios of soda to soap-dust are 2 : 3 and 4 : 5 respectively. If 10 kg of first type be mixed with 18 kg of the second, calculate the amount of soap-dust in the mixed detergent powder.

16. $\frac{1}{3}$rd and $\frac{1}{4}$th part of two pots of equal volume are filled with fruit juice. Water is now added to the pots to full capacity and the two are mixed in a third pot. What is the ratio of fruit juice to water in the new pot ?

17. Three glasses of same volume are filled with beverage. The ratio of water to syrup in the 3 glasses are 3 : 1, 5 : 3 and 9 : 7 respectively. The contents of the three glasses are mixed in a large vessel. What is the present ratio of water to syrup ?

18. In a type of german silver the ratio of copper, zinc and nickel is 4 : 3 : 2. How much zinc should be added to 54 kg of such german silver to make the ratio 6 : 5 : 3 ?

19. A peasant used 22 sackful cowdung for cultivating radish. He also used 12 sackful ammonia and potash in the ratio 2 : 1. Find the ratio of the three types of fertilizers used.

20. In two types of brass the ratios of copper to zinc are 8 : 3 and 15 : 7 respectively. If the two types of brass be mixed in the ratio 5 : 2 a new type of brass is obtained. Find the ratio of copper to zinc in this new type of brass.

21. In two types of stainless steel the ratios of chromium and steel are 2 : 11 and 5 : 21 respectively. In what proportion should the two types be mixed so that the ratio of chromium to steel in the mixed type be 7 : 32 ?

22. The expenses for rice and wheat for a family is 3 : 2 when the ratio of the market rates of the respective items is in the ratio 2 : 1; what will be the ratio of expenses for the same when their market rates are in the ratio 5 : 2. It is to be assumed that the family consumes same quantities fo rice and wheat all the time.

CHAPTER 2

Percentage

In the previous chapter you have learnt to find the proportional parts of the components (ingredients) of the mixture consisting of two or more ingredients. For example:

If in one kind of mixed fertilizer the ratio of urea to potash is 3 : 2, then the proportional parts of the ingredients of this mixed feritllizer are

$$\text{Urea} = \frac{3}{5}; \qquad \text{Potash} = \frac{2}{5}$$

It follows from this that, one part of mixed fertilizer contains $\frac{3}{5}$ parts of urea and $\frac{2}{5}$ parts of potash or 5 parts of mixed fertilizer contain of 3 parts of urea and 2 parts of potash.

We know that if both the numerator and the denominator of a fraction are multipled or divided by the same number the fraction remains unchanged.

So if we multiply the numerators and the denominators of $\frac{3}{5}$ and $\frac{2}{5}$ by 100 respectively, we get

$$\text{urea} = \frac{3 \times 100}{5 \times 100} = \frac{60}{100}, \quad \text{Potash} = \frac{2 \times 100}{5 \times 100} = \frac{40}{100}$$

$\therefore \frac{60}{100}$ and $\frac{40}{100}$ are the proportional parts of urea and potash in the mixed fertilizer i.e., 100 parts of mixed fertilizer contain 60 parts of urea and 40 parts of potash.

Now this method by which we express the idea of "so much in 100" is known as "percentage" and the number expressed in this way is known as "percentage part" or "rate percent".

$\therefore$ urea $= \frac{60}{100}$ parts = 60 parts in 100 = 60%

and potash $= \frac{40}{100}$ parts = 40 parts in 100 = 40%

The mathematical symbol for percent is "%"

One point it to be noted : Percentage always expresses the parts out of 100 or it is a fraction whose numerator is that number and the denominator is 100. Thus you get a technique by which a percentage can be expressed as a fraction. For example :

$30\% = \frac{30}{100}, 18\% = \frac{18}{100}, 25\% = \frac{25}{100}$

In the above example 60% urea means $\frac{60}{100}$ parts of urea, i.e., in 100 parts of the total mixture there is 60 parts of urea.

Therefore, from this you can find the ratio of urea to the total mixture.

i.e., $\frac{\text{Urea}}{\text{total mixture}} = \frac{60}{100}$

in other words, Urea : Total mixture = 60 : 100 = 3 : 5

Similarly, Potash : Total mixture = 40 : 100 = 2 : 5

Again from tis actual ratio of urea to potash can also be found out.

$\frac{\text{Urea}}{\text{Total mixture}} : \frac{\text{Potash}}{\text{Total mixture}} = \frac{3}{2} : \frac{2}{5}$

Or, Urea : Potash = 3 : 2

Again, Urea : Potash = 60% : 40% = $\frac{60}{100} : \frac{60}{100} = 3 : 2$

To express a fraction in percentage :

Suppose $\frac{1}{5}$th part of a stick is under the ground. We are to express this in terms of percentage.

You know $\frac{1}{5} = \frac{1 \times 100}{5 \times 100} = \frac{20}{100} = 20\%$

$\therefore$ 20% of the stick is under the ground.

We can do it by the following method also,

$$\frac{1}{5} = \frac{\frac{1}{5} \times 100}{100} = \frac{20}{100} = 20\%$$

i.e. if a fraction is multiplied by 100 we get the percentage.

e.g. $\frac{3}{4} = \left(\frac{3}{4} \times 100\right)\% = 75\%$

Example 1. What percent is Rs. 15 of Rs. 25 ?

Expressing Rs. 15 as a fraction of Rs. 25 you get $\frac{15}{25} = \frac{3}{5}$

Expressing the fraction $\frac{3}{5}$ in percentage we get

$$\frac{3}{5} \times 100\% = 60\%$$

Example 2. Express 0·5 part of Rs. 25 as a percentage of Rs. 30.

$$0{\cdot}5 \text{ part of Rs. } 25 = \text{Rs. } 25 \times \frac{1}{2} = \text{Rs. } 12{\cdot}5$$

Now, expressing Rs. 12·5 as a part of Rs. 30 we get $\frac{12{\cdot}5}{30} = \frac{5}{12}$

Expressing the fraction $\frac{5}{12}$ in percentage we get

$$\frac{5}{12} \times 100\% = 41\frac{2}{3}\%$$

Application of percentage :

We apply percentage to problems of our daily life, e.g., in statistics, fluctuation in production or in other fields, in calculating interest, in problems of profit and loss, etc.

Application of percentages are shown in the followin examples :

Statistics :

Example 3. 12% of the total population of a village are school students. If the total population of the village is 1225, how many school students are there?

$$\text{The meaning of } 12\% \text{ is } \frac{12}{100}$$

$$\therefore \quad \frac{12}{100} \text{ of } 1225 = 147$$

$\therefore$ 147 are school students.

By the method of proportion :

Let the no. of students be x.

12% may be expresserd as $\frac{12}{100}$ or 12 : 100

Now, this proportion is the same as the proportion of no. of students to the total population of the village.

$$\therefore 12 : 100 = x : 1225$$

$$\therefore \; x = 1225 \times \frac{12}{100} = 147$$

Fluctuations :

Example 4. The production of a land has increased 40% due to the application of chemical fertilizer. If it had produced 35 quintals of paddy in the past, how much paddy will be produced at present ?

40% increase means that if the production was 100 quintals in the past, the present production is (100 + 40) quintals = 140 quintals.

Past Production (quintals)	Present Production (quintals)
100	140
35	x

$$\therefore \quad x = 140 \times \frac{35}{100} = 49$$

Therefore, now the total production of paddy in this land is 49 quintals.

Example 5. The expense of a family on food is 78% of the total monthly expenditure. The rest of the expenditure incurs on house rent. If the house rent is Rs. 187, find the monthly expenditure of the family.

If other expenses except house rent is 78%, then the house rent is (100 – 78) % = 22%

$\therefore$ 22% of the total expenditure is Rs. 187.

i.e. $\frac{22}{100}$ of the total expenditure is Rs. 187.

$$\therefore \text{total expenditure} = \left(\text{Rs. } 187 \times \frac{100}{22}\right)$$

$$= \text{Rs. } 850$$

Again, note the following :

Total expenditure (in rupees)	House rent (in rupees)
100	22
x	187

$$\therefore x = 100 \times \frac{187}{22} = 850$$

Therefore, total expenditure = Rs. 850

Example 6. In a glass factory 1,500 bottles were produced daily. Now the production of bottles is 1,695 daily. By what percent has the production increased ?

Total increase in production = (1695 – 1500) bottles = 195 bottles.

Past Production (No. of bottles)	Present Production (No. of bottles)
1500	195
100	x

$$\therefore \; x = 195 \times \frac{100}{1500} = 13$$

Therefore, rate of increase of production is 13%.

Example 7. The monthly income of a family has increased by 10% due to the increment of salary of the head of the family. But due to the rise in prices of different articles the expenditure has increased by 12%. In the past the income of the head of the family was Rs. 725 and the savings excluding the expenditure was Rs. 25. How much will he save now ?

Family expenditure was Rs. (725 – 25) = Rs. 700.

Due to 10% increment, the salary of the head of the family has become Rs. $\left(725 \times \frac{110}{10}\right)$ = Rs. 797˙50.

Due to 12% increase in expenditure, the expenditure has become Rs. $\left(700 \times \frac{112}{100}\right)$ = Rs. 784.

Present savings will be Rs. (797˙50 – 784) = Rs. 13˙50.

Example 8. If high yeilding variety of seed is used, the production of paddy will increase by 55%. But due to this, the expenses for cultivation increases by 40%. What will be the increase or decrease of an owner of a plot of land which in the past yielded Rs. 640 as against an expenditure of Rs. 250 ?

Due to 55% increase in the production, the production will worth Rs. $\left(640 \times \frac{135}{100}\right)$ = Rs. 992.

Due to the increase in the expenditure the expense for cultivation will be Rs. $\left(250 \times \frac{140}{100}\right)$ = Rs. 350.

In past the income excluding the expenses was Rs. (640 – 250) = Rs. 390.

Now the income excluding the expenses is Rs. (992 – 350) = Rs. 642.

Therefore, increase in income = Rs. (642 – 390)

= Rs. 252.

Exercise 7

1. Express the following percentages in vulgar fractions in their simplest forms :

(*i*) 25% (*ii*) 40% (*iii*) 78% (*iv*) $16\frac{2}{3}\%$

(*v*) $62\frac{1}{2}\%$ (*vi*) $6\frac{1}{4}\%$ (*vii*) $3\frac{1}{3}\%$ (*viii*) $3\frac{1}{8}\%$

2. Convert the following percentages to decimal fractions :

(*i*) 35% (*ii*) 28% (*iii*) 50% (*iv*) 7 %

(*v*) $31\frac{1}{4}\%$ (*vi*) $41\frac{2}{3}\%$ (*vii*) $78\frac{8}{11}\%$

3. Convert the following fractions to percentages :

(*i*) $\frac{1}{2}$ (*ii*) $\frac{3}{8}$ (*iii*) $\frac{37}{40}$ (*iv*) $3\frac{1}{5}$

(*v*) $7\frac{1}{20}$ (*vi*) 0·03 (*vii*) 0·7 (*viii*) 4·4 (*ix*) ·75

4. Evaluate the following :

(*i*) 8 % of Rs. 125 (*ii*) 2 % of 8 km.

(*iii*) 40 % of 65 kg. (*iv*) $43\frac{4}{5}\%$ of Rs. 500.

5. (i) What percent is Rs. 1·25 of Rs. 5 ?
   (ii) What percent is 65 gms of 2 kg?
   (iii) What percent is Rs. 94·14 of Rs. 125·52 ?
   (iv) What percent is 2 kg 250 gms of 0·72 quintals ?

6. The ratio of urea and potash in a mixed fertilizer is 7 : 3. Express the quantities of urea and potash present as percentages of the total amount of fertilizer.

7. The ratio of hydrogen to oxygen in water 2 : 1. Express, the volumes of oxygen and hydrogen present, in percent.

8. Gun-powder consists of sulphur, potash and charcoal in the ratio 2 : 3 : 5. Find the percentages of sulphur, potash and charcoal present in the gun-powder..

9. In a village 30% of the population is literate. If the total population of the village is 6540, how many of them are literate ?

10. In a city 18% of the population are in service. If the total population of the city is 85,100, how many are in service ?

11. There are 840 students in a school, 5% of them are from the minority section. How many students are not from the minority section ?

12. Out of 9160 bighas of land, 15% is irrigated. How much land is non-irrigated ?

13. If high yeilding variety of paddy seed is used, production increases by 20%. How much more of paddy will be produced, if high yielding seeds are sown on a land which generally produces 35 quintals of paddy ?

14. In a certain type of brass, 25% is zinc and the rest is copper. How much copper is present in 264 kg of this quality of brass ?

15. 56, 64 and 80 boys appeared for an examination from a school in three consecutive years and 75%, 87·5% and 93·75% passed respectively in the three years. What is the net percentage of pass in the three years ?

16. In a certain area in Howrah 495 persons are educated out of a total of 1500. What is the percentage of educated people in the area ?

17. Out of his income Mihir Ghosh spends 18% on house rent, 8% on education and saves 5%; the rest is spent for other family expenses. If his monthly income is Rs. 1925, how much does he spend on family expenses?

18. The distance from Howrah to Burdawn via chord line is 85 km. and the distance is 5% more by main line. What is the distance in kilometre by main line ?

19. In ordinary air the volumes of nitrogen, oxygen and carbon-di-oxide gases present are 75·6%, 23·04% and 1·36%. What are the volumes of the gases in 25 litres of air ?

20. An agricultural co-operative distributes its yearly income in the following way : 12% is kept aside for next year's cultivation cost, 3% is spent for the village school and health centre, 5% for development of roads and irrigation, 8% for reserve fund and the rest is equally distributed among its 108 member - families. If in a year each family receives Rs. 6,000 then what are the expenses towards other heads ?

21. The production of a workshop manufacturing electrical machine parts increases from Rs. 56,000 to Rs. 75,600. By what percent does the production increase ?

22. In a co-operative with two handloom cloth of worth Rs. 6,250 was produced every month. Two powerlooms were installed there. If the production rate of the powerlooms is 64% higher than that

of handlooms (i) what is the monthly production of the powerlooms ? (ii) What is the total monthly production at present ? (iii) by what amount the net percentage does the production increase due to installation of powerlooms ?

23. The ratio of bus and train fares from Calcutta to a certain place is 2 : 3. If train fare increases by 20% and the bus fare by 10%. What will be the new ratio of the fares ?

24. During a recent flood $31\frac{1}{4}$ % of the population of a certain town left it ; but 10,560 persons did not leave. What was the propulation of the town before flood ?

25. Using high yeilding variety the production of paddy increases by 30%, but the cost of production also increases by 35%. By how much does the income of a peasent increases from a land which earlier produced paddy worth Rs. 1,220 against an investment of Rs. 450 ?

26. The price of coal rises by 25%. If a family wants to keep its expense for coal same, by what ratio should it decrease its coal consumption ?

27. A Panchayet Samity spends 40% of its Government grant for health and 25% for education. If the expenses on health be Rs. 2,100, then what is the net amount of grant and what is the expense on education ?

28. Out of 1,500 bighas of land a co-operative agricultural farm land allotted for the cultivation of paddy, wheat and jute is in the ratio 7 : 3 : 2. The members of the co-operative decided to increase land for cultivating wheat by 56% and it should be done by taking equal amount of land from the land allotted for cultivation of paddy and jute. Then at present (i) how much of land is for wheat cultivation ? (ii) by what per cent is the land for paddy cultivation reduced, (iii) by what per cent is the land for jute cultivation reduced, (iv) what is the ratio of the land used for cultivation of paddy, wheat and jute in the new system ?

29. In a certain school 85% passed in Bengali, 70% in mathematics and 65% passed in both the subjects. If the total number of examinees be 60, then how many of them (i) passed in both the subjects, (ii) passed only in Bengali (iii) passed only in mathematics and (iv) failed in both the subjects ?

30. In a certain assembly constituency election, 80% of voters exercised their voting right and the winning candidate got elected with 65% of votes polled. What percent of total votes did he poll ?

31. In a particular type of stainless steel the percentage of iron and chromium is 85% and 15% respectively; what are the amounts of iron and chromium in 10 kg of steel ? How much chromium should be added to the steel so that the percentages of iron and chromium become 80% and 20% respectively ?

32. The speed of electric train is 25% more than that of steam engine train. What is the time taken by an electric train to cover a distance which a steam engine train takes 4 hours 25 minutes to cover ?

33. Owing to the rise of price of petrol the transportation cost of goods increases by 15%. What will it now cost to send a quantity of goods from Calcutta to Siliguri which previously costs Rs. 70 ?

34. By statistical investigation in an area a Panchayet Samity obtained the following data :

Subject	1978			1983		
	Female	Male	Total	Female	Male	Total
Total population	7,200	7,625	14,825	8,100	8,235	16,335
Educated population	864	2,745	3,609	1,134	3,294	4,428
Total no. in service	1,328			3,267		
Total no. of agriculturists	3,558			3,231		
Total no. of field labourers	5,337			6,534		
Total no. of unemployed	4,602			3,303		
Road	2·5 km			2·9 km		
Irrigation canal	1·6 km			1·76 km		

Calculate and write in your exercise book the appropriate number that will have to be inserted in the blank spaces in the following statements on the basis of the above data.

(i) In the year 1978 the percentage of the educated among women was less than that among men by .... % and in 1983 it was less by .... %.

(ii) In the year 1978 illiteracy was .... % (approximately) and in 1983 it was .... % (approximately).

(iii) In the year 1983 .... % of total population was employed.

(iv) In the year 1978 the number of agricultural labourers were ..... % of total population, in 1983 it became .... % and the annual rate of increase is .... %.

(v) From the year 1978 to 1983 road increases by .... %.

(vi) From the year 1978 to 1983 irrigation canal increases by .... %.

CHAPTER 3

Problems leading to Harder Fraction

Example 1. A man covers $1\frac{2}{3}$ and $1\frac{1}{6}$ portions of $3\frac{1}{2}$ parts of the distance from his house to a certain destination by train and bus respectively and the rest on foot. What portion of the total distance does he cover on foot.

In mathematical language the problem is,

$$1-\left(\frac{1\frac{2}{3}}{3\frac{1}{2}}+\frac{1\frac{1}{6}}{3\frac{1}{2}}\right)=?$$

In class VI you have seen complex fractions. So you see it is a problem of complex fractions. Now let us see how this type of problems can be solved.

$$\begin{aligned}1-\left(\frac{1\frac{2}{3}}{3\frac{1}{2}}+\frac{1\frac{1}{6}}{3\frac{1}{2}}\right). &=1-\left\{\left(1\frac{2}{3}\div 3\frac{1}{2}\right)+\left(1\frac{1}{6}\div 3\frac{1}{2}\right)\right\}\\ &=1-\left\{\left(\frac{5}{3}\times\frac{2}{7}\right)+\left(\frac{7}{6}\times\frac{2}{7}\right)\right\}\\ &=1-\left(\frac{10}{21}+\frac{1}{3}\right)=1-\left(\frac{10+7}{21}\right)\\ &=1-\frac{17}{21}=\frac{21-21}{21}=\frac{4}{21}\end{aligned}$$

Therefore, the man walked $\frac{4}{21}$ portion of the distance.

Example 2. A housing Co-operative purchased some land. It kept aside $1\frac{1}{8}$ portion of $5\frac{1}{2}$ parts of total land for road. $\frac{1}{5}$ portion of $1\frac{3}{4}$ parts of the remaining land was kept aside for a park and for establishing a public educational and cultural centre. The rest of the land was divided into 32 equal plots and was distributed among the members.

The amount of land in each plot was 2.5 kathas.

i) How much land was purchased by the Co-operative ?

ii) How much land was kept aside for road ?

iii) How much land was kept aside for park, and educational and cultural centre ? [20 *Kathas* = 1 *Bigha.*]

Let the amount of Land purchased by the Co-operative be 1.

$\therefore$ Amount of land kept aside for road $= \dfrac{1\frac{1}{8}}{5\frac{1}{2}}$ of 1

$\therefore$ The amount of land excluding the road

$$= 1 - \frac{1\frac{1}{8}}{5\frac{1}{2}} \text{ of } 1$$

The amount of land kept aside for park, and educational and cultural centre $= \dfrac{\frac{1}{5}}{1\frac{3}{4}}$ of $\left\{1 - \left(\dfrac{1\frac{1}{8}}{5\frac{1}{2}} \text{ of } 1\right)\right\}$

The amount of land each member will get

$$= \left[1 - \frac{1\frac{1}{8}}{5\frac{1}{2}} \text{ of } 1 + \frac{\frac{1}{5}}{1\frac{3}{4}} \text{ of } \left\{1 - \left(\frac{1\frac{1}{8}}{5\frac{1}{2}} \text{ of } 1\right)\right\}\right] \div 31$$

Now the problem is

How much is the total amount of land if

$$\left(\left[1-\frac{1\frac{1}{8}}{5\frac{1}{2}}\text{ of }1+\frac{\frac{1}{5}}{1\frac{3}{4}}\text{ of }\left\{1-\left(\frac{1\frac{1}{8}}{5\frac{1}{2}}\text{ of }1\right)\right\}\right]\div 31\right)$$

of the total land is 2.5 kathas ?

∴ Total amount of land in unit of katha

$$=\frac{2{\cdot}5}{\left[1-\frac{1\frac{1}{8}}{5\frac{1}{2}}\text{ of }1+\frac{\frac{1}{5}}{1\frac{3}{4}}\text{ of }\left\{1-\left(\frac{1\frac{1}{8}}{5\frac{1}{2}}\text{ of }1\right)\right\}\right]\div 31}$$

$$=\frac{2{\cdot}5}{\left[1-\left\{\frac{9}{44}+\frac{4}{35}\times\frac{35}{44}\right\}\right]}$$

$$=\frac{2{\cdot}5}{\left[1-\left\{\frac{9}{44}+\frac{4}{44}\right\}\right]\div 31}$$

$$=\frac{2{\cdot}5}{\left[1-\frac{13}{44}\right]\div 31}$$

$$=\frac{2{\cdot}5}{\frac{1}{44}}=110$$

Therefore, (i) Total amount of land = 110 Kathas = 5·5 Bighas.

(ii) The amount of land kept aside for road

$$=\left(\frac{1\frac{1}{8}}{5\frac{1}{2}}\text{ of }110\right)\text{Khathas}=\left(\frac{9}{44}\text{ of }110\right)\text{Cottahs}$$

= 22·5 Kathas.

(iii) The amount of land kept aside for park, and educational and cultural centre.

$$=\left[\frac{\frac{1}{5}}{1\frac{3}{4}} \text{ of } \left\{1-\left(\frac{1\frac{1}{8}}{5\frac{1}{2}} \text{ of } 1\right)\right\}\right] \text{ of } 110 \text{ Kathas}$$

$$=\frac{1}{11} \text{ of, } 110 \text{ Cottahs} = 10 \text{ Kathas}$$

Example 3. A person deposited his savings with a bank at 10% simple interest. In his will he expressed that after his death the amount with interest be divided into $3\frac{1}{2}$ parts, out of which $1\frac{1}{2}$ portion would go to his wife and the rest would be given to his three sons in equal share. When he died after 4 years from the date of deposit, each of his sons received Rs. 5000. How much did he deposit ?

Let us suppose that the deposited Re 1.

∴ after four years the amount will be

$$\text{Rs.}\left(1+\frac{1}{10}\times 4\right).$$

Now as per the conditions of the will the amount to be received by his wife.

$$= \text{Rs.}\left\{\frac{1\frac{1}{2}}{3\frac{1}{2}} \text{ of } \left(1+\frac{1}{10}\times 4\right)\right\}$$

∴ The remaining amount

$$= \left[\left(1+\frac{1}{10}\times 4\right)-\left\{\frac{1\frac{1}{2}}{3\frac{1}{2}} \text{ of } \left(1+\frac{1}{10}\times 4\right)\right\}\right]$$

∴ the share of each son

$$= \left[\left(1 + \frac{1}{10} \times 4\right) - \left\{\frac{1\frac{1}{2}}{3\frac{1}{2}} \text{ of } \left(1 + \frac{1}{10} \times 4\right)\right\}\right] \div 3$$

Therefore, according to condition the problem stands as

$$\text{If} \left(\left[\left(1 + \frac{1}{10} \times 4\right) - \left\{\frac{1\frac{1}{2}}{3\frac{1}{2}} \text{ of } \left(1 + \frac{1}{10} + 4\right)\right\}\right] \div 3\right)$$

is Rs 5,000, then what is the amount of deposit ?

∴ Amount of deposit

$$= \frac{5000}{\left[\left(1 + \frac{1}{10} \times 4\right) - \left\{\frac{1\frac{1}{2}}{3\frac{1}{2}} \text{ of } \left(1 + \frac{1}{10} \times 4\right)\right\}\right] \div 3}$$

$$= \frac{5000}{\left\{\left(1 + \frac{2}{5}\right) - \left(\frac{3}{2} \times \frac{2}{7}\right)\left(1 + \frac{2}{5}\right)\right\}} \div 3$$

$$= \frac{5000}{\left\{\frac{7}{5} - \left(\frac{3}{7} \times \frac{7}{5}\right)\right\} \div 3}$$

$$= \frac{5000}{\left(\frac{7}{5} - \frac{3}{5}\right) \div 3} = \frac{5000}{\frac{4}{5} \times \frac{1}{3}}$$

$$= \frac{5000}{\frac{4}{15}} = 18750$$

∴ The whole portion is Rs. $5000 \times \frac{15}{4}$ = Rs. 18,750

∴ The man deposited Rs. 18,750 in the Bank.

Example 4: Simplyfy : $\frac{5}{1\frac{1}{3} \text{ of } 1\frac{7}{8}} \times 1\frac{1}{4} \text{ of } \frac{2\frac{1}{10}}{3\frac{1}{2}}$.

$$\frac{5}{1\frac{1}{3} \text{ of } 1\frac{7}{8}} \times 1\frac{1}{4} \text{ of } \frac{2\frac{1}{10}}{3\frac{1}{2}} = \frac{5}{\frac{4}{3} \text{ of } \frac{15}{8}} \times \frac{5}{4} \text{ of } \left(\frac{21}{10} \times \frac{2}{7}\right)$$

$$= \frac{5}{\frac{5}{2}} \times \frac{5}{4} \text{ of } \left(\frac{21}{10} \times \frac{2}{7}\right)$$

$$= 2 \times \left(\frac{5}{4} \times \frac{3}{4}\right)$$

$$= \frac{3}{2} = 1\frac{1}{2}$$

Exercise 8

1. Simplify :

(*i*) $5\frac{7}{8} - 1\frac{3}{4} + 3\frac{5}{6} - 4\,\frac{1}{12}$

(*ii*) $1 \div [1 + 1 \div \{1 + 1 \div (1 + 1 \div 2)\}]$

(*iii*) $\frac{19}{21} \div [\frac{2}{3} - \{\frac{5}{7} - (4\frac{1}{2} - \frac{1}{3} \div \frac{1}{4} - \frac{1}{6})\}]$

(*iv*) $\frac{2\frac{2}{3} + 5\frac{7}{9}}{1\frac{1}{2} - \frac{4}{9}} \div \left(3\frac{1}{2} \text{ of } \frac{5}{8}\right) \times \frac{2\frac{3}{6}}{32}$

(*v*) $\left(2\frac{3}{4} + \frac{2\frac{1}{2}}{3\frac{4}{5}} \text{ of } 7 - \frac{1\frac{2}{3}}{2\frac{1}{2}}\right) \quad \left(2 - \frac{151}{228}\right)$

(*vi*) $13\frac{9}{19} \div \frac{1}{416 \times 1{\cdot}2625} \text{ of } \qquad 7\frac{5}{57}$

$$\frac{3+\dfrac{21}{7\frac{3}{5}}\ \text{of}\ \frac{5}{3}-\frac{1}{4}-\dfrac{1\frac{2}{3}}{2\frac{1}{2}}}{10-\dfrac{151}{228}\text{of } 5}$$

(*viii*) $\dfrac{\frac{2}{3}\div\frac{5}{6}\ \text{of}\ \frac{3}{4}}{\frac{2}{3}\div\frac{3}{4}\times\frac{5}{6}}-\dfrac{7{\cdot}7\times0{\cdot}12}{2{\cdot}1}$

Miscellaneous Exercise 1

1. The ratio of $\frac{27}{64}$ with a number is equal to its ratio with $\frac{3}{4}$; find the number.

2. In an alloy, the ratio of copper and zinc is 5 : 2. How much zinc is to be mixed in 17 kg 500 gms of alloy to have copper and zinc in the ratio 2 : 1?

3. The ratio of the present ages of Munna and Runtu is 4 : 3; eight years hence the ratio of their ages will be 16 : 13. What are their present ages?

4. In a field the ratio of yeilds before and after irrigation facilities are provided is 4 : 7. What will be the production for a plot of land in that field, if the production before irrigation facilities are provided is 15·2 quintals?

5. Following are the quantities showing oil yields from different oil seeds and coconut:

Ground nut	45%	Til	44%	Kusumseed	30%
Mustard	33%	Linseed	36%	Coconut	60%

From the above data solve the following problems .

(*i*) Find the ratio of the oil yields from :

(*a*) Ground nut and Coconut

(*b*) Mustard and Til

(*c*) Kusumseed and Mustard

(*d*) Linseed and Ground nut

(*ii*) How much oil can be obtained from the Coconut, the quantity of which is equal to the quantity of Ground nut yielding 12 kg of oil?

(*iii*) How much Linseed is required so as to have the quantity of oil equal to that yielded from 25 kg of Til? (obtain the approximate value correct upto two places of decimals).

6. The sum of the present ages of father and his son is 65 years. Ten years ago the ratio of their ages was 7 : 2. What will be the ratio of their ages ten years hence?

7. Arun, Shyamal and Rasid raised capitals of Rs. 10,000, Rs. 8,000 and Rs. 12,000 respectively and decided to purchase a bus. They got a bank loan of Rs. 75,000 on condition that every year they would repay Rs. 15,000 as loan and Rs. 2,500 as

interest. Arun worked as driver and Shyamal and Rasid worked as conductors. For distribution of profit it was proposed, that after repayment of the bank loan, one-third of the remaining amount would be divided the ratio of their respective capitals and the rest in the ratio 3 : 2 : 2. If the net profit in a particular year be Rs. 49,000, what amount will each of them get?

8. In a box there are 378 coins in one rupee, 50 paise and 25 paise coins. If the ratio of their values be 13; 11 : 7, find the number of each kind of coins in the box.

9. Two glasses contain water mixed with syrup. The ratio of the volumes of two glass containers is 3 : 5 and syrup and water in the mixture are in the ratio 1 : 8 and 2 : 13 respectively. If the contents of the two are mixed in a large vessel, what will be the ratio of syrup and water in the new mixture? How much syrup would have to be mixed with 2 litres of the new mixture to make the ratio of syrup and water as 1 : 5?

10. A 25 H.P. pump can lift 7,000 litres of water per minute from a well 250 metres deep. What will be the H.P. of a pump that can lift 4,200 litres of water per minute from a well of 200 metres deep?

11. To extend irrigation facilities in their field, 120 villagers of Natungram started digging a feeder canal. After 7 days, 8 of them fell ill. In the mean time, they have completed $\frac{5}{7}$th part of the work. How many more days would they require to complete the whole work?

12. An iron plate 15 dcm long, 7·5 dcm wide and 1·5 dcm thick weighs 12 tons. What is the weight of a peice of the same plate, 8 dcm long and 3 dcm wide?

13. 15 workmen make a contract to dig a pond in 28 days. They start working 6 hours a day and after 18 days it is found that $\frac{6}{11}$ part of the work is done. How many hours a day shall they work from now to finish the work in time?

14. The area of a plot of land is 1,176 square metres. The ratios of length and breadth of another plot to those of the former are 3 : 4 and 2 : 3 respectively. What is the area of the other plot?

Miscellaneous Exercise 1

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14. The area of a plot of land is 1,176 square metres. The ratios of length and breadth of another plot to those of the former are 3 : 4 and 2 : 3 respectively. What is the area of the other plot?

15. The ratio of the diameters of the front and rear wheels of an engine is 5 : 2. To travel 220 km, the rear wheel revolves 50,000 times. How many revolutions does the front wheel make in the same time? What is the diameter of the front wheel?

16. Fill in the blanks :

	Vulgar fraction		Decimal fraction		Percentage
(*i*)	$\frac{2}{3}$	=	....	=	....
(*ii*)	$\frac{1}{6}$	=	....	=	....
(*iii*)	$\frac{2}{20}$	=	....	=	....
(*iv*)	....	=	0·035	=	....
(*v*)	....	=	0·04	=	....
(*vi*)	....	=	0·008	=	....
(*vii*)	....	=	....	=	$\frac{3}{4}$
(*viii*)	....	=	....	=	$12\frac{1}{2}$
(*ix*)	....	=	....	=	11
(*x*)	....	=	....	=	17·5

17. Find the values of :

(*i*) 2% of Rs. 175 (*ii*) 20% of 625 gms (*iii*) 1·5% of 450 m (*iv*) 0·6% of 600 km (*v*) $13\frac{1}{2}$% of 12·9 sq. cm (*vi*) 5% of Rs. 25·40 (*vii*) 120% of 120 tons (*viii*) 1% of 1.

18. (*i*) What percentage of 40 m of cloth would you take to have 7 m of cloth?

(*ii*) What percentage of 0·2 km must one travel to travel 25 m?

(*iii*) 80% of a number is 30; find the number.

(*iv*) 1% of the roll-strength of a school is 8. Find the number of students in the school.

19. The following chart shows some data collected by a village co-operative society in their area every fifth year :

Subject	1965	1970	1975	1980
Population	15,000	16,500	18,150	19,600
Literates	3,000	3,630	4,356	4,500
Service holders	1,800	1,650	2,145	2,550
Cultivators	3,750	4,125	3,630	3,450
Craftsmen	2,250	2,475	2,178	1,960
Agricultural Labour	5,250	6,105	7,260	7,820
Unemployed	1,050	1,650	2,178	2,200

Use the above data to answer the following questions: (Find the result correct upto 3 places of decimals)

(*i*) What is the yearly growth percent of population from 1965 to 1980?

(*ii*) In which year was the percentage of literates highest?

(*iii*) What is the growth rate per cent of service holders from 1965 to 1980? What is the annual growth rate? Compare this result with that of population?

(*iv*) What is the ratio of service holders to cultivators in 1970? What was it in 1980? What is the percentage of the former to the latter?

(*v*) What is the growth or fall in rate per cent of cultivators between 1965 and 1980?

(*vi*) Find the ratio of cultivators to agricultural labours in 1975. What was the ratio in 1980? What was the percentage of the former to the latter?

(*vii*) What is the percentage of unemployed in 1980? Is it greater than that in 1970? If so, then by what per cent?

20. In the last annual examination Sibani stood first by securing 68% marks, whereas Mausumi secured second position securing 0·5% marks less than Sibani. If Mausumi secured 540 marks, what was the aggregate marks in the examination? What was the percentage and how much did Sibani secure?

21. The new irrigation system has increased yeild of a particular field on an average by 45%. Previously, Budhai Majhi used to yeild 15 quintals of paddy from a particular plot of land in that field. What will be his yeild now? If he is to pay the price of 1·5 quintals of paddy as irrigation tax, what will be the increase in his income in terms of price of paddy and what is the percentage?

22. An agricultural co-operative allots its total land in the ratio 7 : 2 for cultivation of paddy and wheat respectively. In this year 177 bighas of land are allotted for sugar-cane cultivation and the land for wheat cultivation is increased by 25% and the rest is allotted for paddy cultivation. If total land of the co-operative is 1,350 bighas, then

(*i*) how many bighas of land were for paddy cultivation earlier?

(*ii*) how many bighas of land are now allotted for paddy cultivation? Calculate the decrease in percentage?

(*iii*) what percentage of the total land is now allotted for sugar-cane cultivation?

(*iv*) paddy : wheat : sugar-cane = ? (in respect of land)

(*v*) next year if another 52·5 bighas of land are allotted for wheat cultivation, by what per cent will it increase with respect to the previous year?

23. In the last annual examination 20%, 45% and 30% of the students of a school got plucked in Bengali, English and Mathematics respectively and 15% got plucked in all the three subjects, but none got plucked in two subjects only. If the total number of students be 560, how many of them did pass in all the subjects?

24. 68% of the population of a village confined by flood were women and children. On the first night 80% of them were removed to a safe place. Next morning it was found that 85 persons more are to be removed in order to make all women and children safe.

(*i*) What was the total population of the village?

(*ii*) How many persons were remained in the village when all the women and children were removed to the safe place?

(*iii*) How many persons were removed on the first night?

25. In a particular constituency 75% of the voters cast their votes, out of which 2% were rejected. The winning candidate received 75% of the valid votes and bagged a total of 9,261 votes.

(*i*) What was the total number of voters in the constituency?

(*ii*) How many votes were rejected?

(*iii*) What per cent of total votes did the winning candidate get?

(*iv*) How many valid votes did the defeated candidate get?

(*v*) What per cent was it to the total votes cast?

26. In the last year a weavers' co-operative put aside 15% of its net income as reserve fund and distributed the remaining income among its 34 members, each weaver-member received Rs. 7500 as dividend. In this year the co-operative purchased two new power looms taking loan from a bank. As a result the net income of the co-operative has gone up by 20%. in this year the society has put aside 8·5% of its net income to repay the bank loan in addition to 15% as reserve fund. The rest of the income was distributed among the weaver-members as dividends.

(*i*) How much dividend will each weaver-member receive in this year?

(*ii*) Calculate the ratio of this year's reserve fund to that of the previous year.

(*iii*) How much is set aside for repayments of bank loan?

27. At the time of retirement an employee received some money as provident fund and gratuity. He deposited 25% of that money as fixed deposit with a bank. He purchased a plot of land with 15% of the remaining

portion and the rest was spent to build his house. Now as he lives in his own house, he is relieved of Rs. 250 per month. He finds that investment on house including the land is equivalent to 12·5% of his yearly savings.

(*i*) How much did he receive in cash at the time of retirement?

(*ii*) How much did the plot of land cost?

(*iii*) How much did he spend to build the house?

(*iv*) How much did he deposit with the bank?

CHAPTER 4

INTEREST

In practical life, problems regarding interest arise, when a creditor (*i.e.* a person who lends money) allows the debtor (*t.e.* a person who takes the loan) to utilise his money for a certain period of time. Obviously no one will allow others to use his money without any return. Naturally, at the time of transaction the creditor, with the intention to have some monetary gains, imposes some conditions on the debtor and these conditions come up as different problems on interest.

In lieu of the right for temporary use of creditor's money the debtor is obliged to pay some money. This money is known as 'interest'. At the time of transaction generally both parties agree on two points–time for which the money is lent and the rate of interest to be paid thereof.

Lending money personally against interest is in vogue from the ancient time in mostly all the countries of the world. But it is seen that all cheif faiths of religion have disapproved usury as a practice and even marked usurers as an object of social and religious hatred. But under the existing socio-economic conditions it could not be stopped anywhere. Now-a-days due to the expansion of banking system, personal usury has reduced to certain extent. In modern times with the changed state and society, public opinion is against this extortion of unholy and undue interest. In our country also personal usury has. been declared illegal in the year 1974. So in this chapter we shall only discuss those cases where problems of interest arise in monetary transactions with banks, post offices, co-operative societies and other recognised money lending institutions. Here we shall only deal with the problems of simple interest. In institutions like banks, post offices etc.

interests are claculated in a way which is different from that of simple interest. This complex procedure you will learn later on.

In class VII you have solved problems on simple interest (such as determination of interest, principal, time etc.) using unitary method. But in the mean time you have learnt relative weightage, interaction of things and events along with the concepts of ratio-proportion, percentage, rule of three etc. Here we shall use these concepts to solve varieties of complex problems on interest.

Generally the concepts that occur in problems on interest are :

(i) Principal : It is the amount of loan taken or given or kept as deposit.

(ii) Time : It is the period for which the loan is given or taken or the money is kept as deposit.

(iii) Total interest : It is interest earned or due on a particular principal used for a definite period.

(iv) Amount : Principal + total interest.

(v) Rate of Interest : Interest on Rs. 100 for 1 year. For example, rate of interest 10% means interest on a principal of Rs. 100 for 1 year is Rs. 10. So after one year the amount will be Rs. (100 + 10) = Rs. 110, since here the total interest is Rs. 10.

By rate per cent of interest we mean interest on Rs. 100 for one year. So from it we can easily deduce interest on Re. 1 for one year, interest on Rs. 10 for one year etc.

Relation among the different aspects of interest :

(i) If rate of interest and time remain unchanged : Principal and total interest will be directly related, i.e., if principal increases, total interest will increase if principal decreases, total interest will decrease.

(ii) If principal and time remain unchanged : Rate of interest and total interest will be directly related, i.e., if rate of interest increases, total interest will increase; if rate of interest decreases, total interest will decrease.

(iii) If principal and rate of interest remain unchanged : Time and total interest will be directly related,

i.e., if time increases, total interest will increase; if time decreases, total interest will decrease.

(iv) If rate of interest and total interest remain unchanged : Principal and time will be related inversely, i.e., if principal increases, time will be less and if principal decreases; time will be more.

(v) If principal and total interest remain unchanged : Rate of interest and time will be related inversely, i.e., if rate of interest increases, time will be less and if rate of interest decreases, time will be more.

(vi) If time and total interest remain unchanged : Principal and rate of interest will be related inversely, i.e., if principal increases, rate of interest will be lower and if principal decreases, rate of interest will be higher.

N.B. The above relations will be helpful in obtaining proportions for the purpose of solution of problems on interest using the rule of three.

Determination of Total Interest :

Example 1: What is the total interest on Rs. 750 in 3 years at the rate of 5% simple interest per annum?

In the first part of the problem, we have "at the rate of 5% simple interest per annum", which means principal = Rs. 100, time = 1 year and total interest = Rs. 5. Again in the second part of the problem, we have "what is the total interest on Rs. 750 in 3 years?" which means principal = Rs. 750, time = 3 years and total interest = ? So we can put the problem in language of mathematics as follows :

Principal (in rupees)	Time (in years)	Total interest (in rupees)
100	1	5
750	3	?

From the table taking Rs. x for the required total interest, the problem may be solved with the help of rule of three as :

$$x = 5 \times \frac{3}{1} \times \frac{750}{100} = 112{\cdot}5$$

$\therefore$ required total interest = Rs. 112·50

From the first step of rule of three we can deduce an algebraic formula as follows :

Let us assume :

Principal = P, time = t, total interest = I and rate of interest = i (interest rate per cent ÷ 100, i.e., interest on Re. 1 for 1 year).

In the present problem, $P = 750, t = 3, i = \frac{5}{100}$ and $I = ?$

We can write the first step by using the rule of three

$$\text{Total interest} = 5 \times \tfrac{3}{1} \times \frac{750}{100}$$

$$= 750 \times 3 \times \frac{5}{100}$$

$$= \text{Principal} \times \text{time} \times \text{rate of interest.}$$

So we get the algebraic formula :

$I = P \times t \times i = Pti$

Let us see an application of this algebraic formula :

Example 2: What is the total interest on Rs. 350 in 8 years at the rate of 7% simple interest per annum?

Here $P = 350,\ t = 8,\ i = \frac{7}{100} = 0{\cdot}07,\ I = ?$

So according to the formula,

$I = Pti = 350 \times 8 \times 0{\cdot}07 = 196.$

$\therefore$ The required total interest is Rs. 196.

Remark : For convenience of calculation we shall express i in decimals.

Determination of Time :

Example 3 : In how many years will Rs. 400 produce an interest of Rs. 140 at the rate of 7% simple interest per annum?

In language of mathematics, the problem is :

Principal (in rupees)	Time (in years)	Total interest (in rupees)
100	1	7
400	?	140

Taking x years for the required time and by rule of three we have

$$x = 1 \times \frac{140}{7} \times \frac{100}{400} = 5$$

$\therefore$ The required time is 5 years.

Let us see how we can establish an algebraic formula for it.

We know, $I = Pti$;

hence we get the corollary, $t = \frac{I}{pi}$. *(corollary 1)*

$$\textit{i.e.}, \text{ Time} = \frac{\text{Total interest}}{\text{Principal} \times \text{rate of interest}}$$

$$\text{Here, } t = \frac{140}{400 \times 0{\cdot}07} = 5 \text{ years.}$$

$\therefore$ The required time = 5 years.

Let us see another application of this formula.

Example 4: In how many years will Rs. 150 amount to

Rs. 222 at the rate of 12% simple interest per annum?

Here $P = 150, I = (222-150), i = \frac{12}{100} = 0{\cdot}12, t = ?$

$$t = \frac{I}{pi} \qquad \text{(corollary 1)}$$

$$= \frac{72}{150\times 0{\cdot}12} = 4.$$

∴ The required time = 4 years.

Determination of Principal :

Example 5: What principal will yeild an interest of Rs. 136 in 4 years at the rate of 10% simple interest per annum?

In language of mathematics, the problem is

Principal (in rupees)	Time (in years)	Total interest (in rupees)
100	1	10
?	4	136

Taking Rs. x for the principal and applying rule of three we get

$$x = 100 \times \frac{1}{4} \times \frac{136}{10} = 340$$

The required principal is Rs. 340.

Let us establish the algebraic formula :

You know, I = Pti

$$\text{Hence } P = \frac{I}{ti} \text{ (corollary 2)}$$

$$\text{i.e. Principal} = \frac{\text{Total interest}}{\text{Time} \times \text{rate of interest}}$$

Let us see an application of the formula.

Example 6: What principal will yield an interest of

Rs. 147 in 7 years at the rate of 6% simple interest per annum?

Here $I = 147, t = 7, i = \frac{6}{100} = 0{\cdot}06, P = ?$

According to corollary 2, $P = \frac{I}{ti} = \frac{147}{7 \times 0{\cdot}06} = 350$

Required principal = Rs. 350.

Determination of Rate per cent :

Example 7: At what rate per cent per annum will Rs. 450 produce a total simple interest of Rs. 135 in 5 years?

In the language of mathematics, the problem is

Principal (in rupees)	Time (in years)	Total interest (in rupees)
100	1	?
450	5	135

Taking Rs. x as the annual interest on Rs. 100 and by rule of three, we have

$$x = 135 \times \frac{1}{5} \times \frac{100}{450} = 6$$

∴ Required rate per cent of interest = 6

Let us see how we can establish the algebraic formula. You know $I = Pti$

$$\therefore\ i = \frac{I}{Pt} \qquad \text{(corollary 3)}$$

i.e., rate of interest = $\frac{\text{Total interest}}{\text{Principal} \times \text{time}}$

Let us see a solution applying this formula.

Example 8: At what rate per cent per annum will Rs. 600 amount to Rs. 696 in 2 years?

Here $P = 600$, $I = (696 - 600) = 96$, $t = 2$, $i = ?$

According to corollary 3, $i = \frac{I}{Pt} = \frac{96}{600 \times 2} \doteq 0{\cdot}08$

$\therefore$ Rate or interest $= 0{\cdot}08$

$\therefore$ The required rate p.c. $= 0{\cdot}08 \times 100 = 8$.

Let us now look at different algebraic formulae at a glance.

The symbols used in the formulae are

(*i*) Principal $= P$

(*ii*) Time $= t$

(*iii*) Total interest $= I$

(*iv*) Rate of interest $= i$

It has been said earlier $i = \frac{\text{Rate of interest per cent}}{100}$

Now have a look at the formulae :

(*i*) formula for total interest is $I = Pti$.

(*ii*) formula for principal is $P = \frac{I}{ti}$

(*iii*) formula for time is $t = \frac{I}{Pi}$

(*iv*) formula for rate of interest is $i = \frac{I}{pt}$

N.B. : One has to calculate the rate of interest per cent by multiplying i with 100. For convenience of computation it is desirable that the rate of interest be expressed in decimal fraction.

Determination of principal and total interest :

Example 9 : What principal will amount to Rs. 747 in 6 years at the rate of 11% simple interest per annum? What will be amount of total interest then?

In the language of mathematics, problem is

Principal (in rupees)	Time (in years)	Total interest (in rupees)
100	1	11
?	6	?

Here $P = ?,\ t = 6,\ I = ?,\ t = 0{\cdot}11$.

In the above problem there are two unknowns. So we cannot solve it directly either by rule of three or by algebraic formula. But if we take Rs. x as one of the unknowns, we should be able to solve the problem by applying algebraic formula.

Let us take $P = x$, then the problem reduces to $I = ?$

We know $I = Pti = x \times 6 \times 0{\cdot}11 = {\cdot}66x$

So according to the problem,

$$x + 0{\cdot}66x = 747$$

$$\text{or, } x(1 + 0{\cdot}66) = 747$$

$$\text{or, } x = \frac{747}{1+0{\cdot}66} = \frac{747}{1{\cdot}66} = 450.$$

$\therefore$ Required principal = Rs. 450

and required total interest = Rs. (747 − 450)

= Rs. 297.

This problem can also be solved in the following way :

As there are two unknowns we cannot directly apply unitary method or rule of three; but we can easily find the total interest on Rs. 100 for 6 years and that is, Rs. (6 × 11) = Rs. 66.

Therefore Rs. 100 will amount to Rs. (100 + 66) = Rs. 166 in 6 years.

So the problem now reduces to

Principal (in rupees)	Amount (in rupees)
100	166
?	747

Now taking Rs. x for the principal and applying rule of three, we have

$$x = 100 \times \frac{747}{166} = 450$$

∴ The required principal is Rs. 450 and the total interest is Rs. (747−450) = Rs. 297.

Harder problems on Interest :

Example 10: Of the two friends, one takes a loan of Rs. 750 from a co-operative society at the rate of 8% simple interest per annum and the other, a certain amount on the same day at the rate of 10% simple interest per annum from a commercial bank. After few years when the first friend has repaid his loan with interest by giving Rs. 1110, the second friend has paid Rs. 330 as interest only. What is the amount borrowed by the second friend?

It is clear that there are two parts of the problem. Let us rewrite it in language of mathematics.

First 8% interest

Principal (in rupees)	Time (in years)	Total interest (in rupees)
100	1	8
750	?	1110 – 750

Second 10% interest

Principal (in rupees)	Time (in years)	Total interest (in rupees)
100	1	10
?	?	330

Even though the problem tells about the final settlement of the second friend it is clear that it cannot be worked out directly as there are two unknowns involved in it. But in the first part the only unknown is the time which can be determined directly. Again it is clear from the condition that the time is the same in both parts.

So to solve the problem the steps to be followed are:

(i) At first we are to determine the time in the first part.

(ii) Then we are to put that time in the table for the second part.

(iii) Finally, by solving the second part the required principal can be obtained.

Now let us take Rs. P for the required principal and t years for the time; then the table takes the form

8% interest

Principal (in rupees)	Time (in years)	Total interest (in rupees)
100	1	8
750	t	360

10% interest

Principal (in rupees)	Time (in years)	Total interest (in rupees)
100	1	10
P	t	330

Solution of the first part: $t = \dfrac{I}{P \times i} = \dfrac{360}{750 \times 0{\cdot}08} = 6$

Now putting 6 in place of t and taking P for Principal in the table of the second part, we get

$$P = \frac{I}{i \times t} = \frac{330}{6 \times 0{\cdot}1} = 550$$

That is, the second friend borrowe Rs 550.

Example 11: At the time of retirement, a person received Rs. 25,800 as provident fund and gratuity. He now desires to deposit a part of it with a post office and the rest with a bank such that his yearly income as interest will be Rs. 2,430. If the rate of simple interest of the post office and that of the bank per annum be 9% and 10% respectively, how much should he deposit with the post office and with the bank?

In the language of mathematics, the problem is

9% interest

Principal (in rupees)	Time (in years)	Total interest (in rupees)
100	1	9
?	1	?

10% interest

Principal (in rupees)	Time (in years)	Total interest (in rupees)
100	1	10
?	1	?

Here in both the tables there being two unknowns, none can be solved directly. But we know the total sum. So if we take Rs. x for one, the other will be Rs. $(25{,}800-x)$.

Now in the first case

Total interest

$$= \text{Rs. } (x \times 1 \times 0{\cdot}09)$$

$$= \text{Rs.} \frac{9x}{100}$$

in the second case

Total interest

$$= \text{Rs. } \{(25800 - x) \times 1 \times 0{\cdot}1\}$$

$$= \text{Rs.} \frac{25800-x}{10}$$

So according to the condition of the main problem

$$\frac{9x}{100} + \frac{25800-x}{10} = 2430$$

or, $9x + 258000 - 10x = 243000$
or, $-x = 243000 - 258000 = -15000$
$\therefore x = 15000$

So he is to deposit Rs. 15000 with the post office and Rs. $(25800 - 15000)$ = Rs. 10800 with the bank.

Example 12: A person made a fixed deposit of Rs. 15000 in a bank for 5 years term at the rate of 10% simple interest per annum. But he had to withdraw the total amount after 3 years, so as to meet the expenses for his daughter's marriage. The bank, of course, paid a lower rate of interest as the money was drawn before maturity. For this he received Rs. 3900 less than what he would have got, had he drawn it after 5 years. What is the rate per cent of simple interest per annum allowed by the bank?

In the language of mathematics the problem is

Rate of interest 10%

Principal (in rupees)	Time (in years)	Total interest (in rupees)
100	1	10
15000	5	?

Rate of interest ?%

Principal (in rupees)	Time (in years)	Total interest (in rupees)
100	1	?
15000	3	?

Here in the first part there is one unknown and so we get

Total interest = Rs. $(15000 \times 5 \times 0{\cdot}1)$ = Rs. 7500

That is, if he had drawn after 5 years, he would have received Rs. 7,500 as interest. But as he had drawn the money after 3 years he received Rs. $(7500 - 3900)$ = Rs. 3600 only as interest.

Now putting 3600 for total interest in the second table, we get

$$\text{Rate of interest, } i = \frac{3600}{15000 \times 3} = 0{\cdot}08$$

So the required rate of single interest per annum = (0·08 × 100)% = 8%

Example 13: A person made two fixed deposits with a bank in the names of his two daughters in such a manner that when each would attain 18 years of age he would receive Rs. 12000 from her account. The bank gives interest of 15% on the fixed deposit. The age of his elder daughter being 14 years, he had to deposit Rs. 2700 less in the name of his younger daughter than that of the elder one. (*i*) How much did he deposit for his daughter? (*ii*) What is the present age of his younger daughter?

In the language of mathematics, the problem can be set into the table tables as shown below :

For the elder daughter
at 15% interest

Principal (in rupees)	Time (in years)	Total interest (in rupees)
100	1	15
?	4	?

For the younger daughter
at 15% interest

Principal (in rupees)	Time (in years)	Total interest (in rupees)
100	1	15
?	?	?

Solution of the first part:

Interest on Rs. 100 for 4 years at the rate of 15% simple interest per annum is Rs. (15 × 4) = Rs. 60

∴ after 4 years Rs. 100 amounts to Rs. (100 + 60) = Rs. 160

Therefore, the problem stands like this :

Principal (in rupees)	Amount (in rupees)
100	160
?	12,000

∴ Taking Rs. x for the principal we have,

$$x = 100 \times \frac{12000}{160} = 7{,}500.$$

Therefore he deposited Rs. 7500 in the name of his elder daughter.

Hence by the condition of the problem, he deposited Rs. $(7500 - 2700)$ = Rs. 4800 in the name of his younger daughter.

Now, the new form of the said table is for the younger daughter at 15% interest per annum.

Principal (in rupees)	Time (in years)	Total interest (in rupees)
100	1	15
4800	?	12000 – 4800

Taking t years for the time we have

$$t = \frac{7200}{4800 \times 0{\cdot}15} = 10$$

So the age of his younger daughter = $(18-10)$ years
= 8 years.

(*i*) He deposited Rs. 7500 and 4800 in the names of his older and younger daughters respectively.

(*ii*) The age of his younger daughter was 8 years.

Exercise 9

1. Find the total interest on Rs. 3500 in 6 years at the rate of $7\frac{1}{2}$% simple interest per annum. What amount will be required to repay the principal with interest?

2. To purchase plough and cattle,a small farmer took a loan of Rs. 700 from a village co-operative bank. If the bank charges $6\frac{1}{4}$% simple interest per annum on loans, how much will he have to pay after two years as interest?

3. Taking a loan at the rate of $12\frac{1}{2}$% simple interest per

annum from a bank, an unemployed youth purchased a cycle rickshaw. After 4 years he paid Rs. 1250 as interest only. How much did he borrow?

4. Sakhaldeo Pandey deposited Rs. 1200 with a bank and after few years, received Rs. 1800 from the bank. If the bank pays 10% simple interest per annum, for how many years was the money kept in the bank?

5. Bank charges 15% simple interest per annum on loans whereas co-operative credit society charges only 10% simple interest per annum. If a person takes a loan of Rs. 3500 for 3 years from a society instead of taking it from a bank, how much will he save?

6. Mr. Sushil Bhattacharya deposited Rs. 7000 with a post office. After his death, his wife received Rs. 10,920 from the post office. If the post-office then allows 8% simple interest per annum on the deposit, how many years ago did Mr. Bhattacharya deposit the money?

7. A Government development corporation receives deposit from the public against 12% simple interest per annum. A person, who deposited a sum with it, received a total amount of Rs. 4800 after five years. How much did he deposit?

8. A house-building co-operative society took a loan of Rs. 75,000 from a co-operative bank at a simple interest of $7\frac{1}{2}$% per annum. After few years it paid Rs. 1,03,125 as amount in full settlement of the loan. How long did the society take to repay the loan?

9. In how many years will a sum double itself at the rate of 8% simple interest per annum?

10. Debkumar Chongdar, a press owner, intending to purchase a rotary machine for Rs. 35,000 took a loan of half the cost of the machine from a bank. If after 4 years he had to pay Rs. 25,900 as amount in full settlement of the loan, what is the rate of simple interest per cent per annum charged by the bank?

11. On the same day Amal Roy and Pasupati Ghosh deposited Rs. 2000 each with bank and post office respectively. After 3 years they received Rs. 2360 and Rs. 2480 respectively. what is the ratio of rates of simple interest per cent per annum

allowed by the bank and the post office?

12. To purchase a powerloom, a weavers' co-operative society takes a loan from a co-operative bank on the condition that after every two years it will pay 7% simple interest per annum and Rs. 5000 towards repayment of principal. If the amount of first instalment be Rs. 13,400 what was the sum borrowed by the society?

13. At the time of retirement,an employee receives some money from provident fund and deposited the same in a bank. If the sum amounts to Rs. 16662·50 in five years at the rate of 11% simple interest per annum, how much did he deposit and how much did he receive as interest?

14. Ramu Kurmi took a loan of Rs. 600 from a bank at the rate of $12\frac{1}{2}$% simple interest per annum and Manu Munda took a loan of Rs. 750 from a village co-operative society at the rate of 9% simple interest per annum .What is the ratio of interests they had to pay after 4 years?

15. After the death of a person his sons found that the money which their father had deposited 7 years ago in post office amounted to Rs. 5215. If the post-office had given 7% simple interest per annum on the deposit what sum did the person deposit?

16. At the time of purchasing a tractor an agricultural co-operative society took a loan of Rs. 12,000 from a bank and paid Rs. 18,000 after 4 years as total amount. What is the rate of simple interest per cent per annum charged by the bank?

17. Haren Mondal took a sum as loan from Ilambazar co-operative credit society. After 3 years he had to pay Rs. 356·25 as total amount. If the rate of simple interest is $6\frac{1}{4}$% per annum, what sum did he borrow?

18. An agricultural co-operative society gives agricultural loan to its members at the rate of 4% simple interest per annum, while a bank charges at the rate of 7·4% simple interest per annum. If a farmer becomes member of the society and takes a loan of Rs. 1,250 from it, instead of taking it from the bank, how much will be saved per year as interest?

19. Post office gives simple interest on fixed deposit at the rate

of 11% per annum. If a person needs to get Rs. 12400 after 5 years at the time of his daughter's marriage, what sum should he deposit now?

20. At the time of purchasing a powerloom, a weaver's co-operative society took a loan of Rs. 15,000 from a bank. After 5 years it had to pay in total Rs. 22,125. What is the rate of simple interest per cent per annum charged by the bank?

21. A person purchased some government loan certificates at 14% simple interest per annum, with the intention that after 10 years his wife and son would share the amount in the ratio 3 : 2. After the expiry of the term if the son gets Rs. 6,720, what sum did he invest?

22. At the time of purchasing a tractor a village agricultural co-operative society took a loan of Rs. 60,000 from the district co-operative bank. In addition to ploughing their own field the society earned some money by letting out the tractor. From that income the society paid back the loan with interest in 15 years. If the rate of simple interest charged by the bank were 6·5% per annum, what was the average yearly income earned by the society by hiring out the tractor?

23. A farmer borrowed a sum from an agricultural co-operative society and purchased a pumpset. Just after one year he borrowed an equal sum from a bank. 4 years after taking the first loan he settled the loan in full with the society paying Rs. 6,848. If he had settled the bank loan too on the same date, what amount did he pay? (The rates of simple interest per annum of the society and the bank were 7% and 11% respectively.)

24. A person deposited a part of his savings with a bank and the rest with a post office. His annual income would have been Rs. 50 more had he deposited the whole savings with the bank. The rates of simple interest per annum of the bank and the post office were 10% and 8% respectively and the amount of his savings was Rs. 5,700. What amounts did he deposit with post office and the bank respectively?

25. A person took a loan of Rs. 400 at the rate of $7\frac{1}{2}$% simple interest per annum and another loan of Rs. 400 at the rate of 8% simple interest per annum just after one year. How

many years after the first loan will the interest for both the loans be equal?

26. A person after taking a loan from a bank for 3 years at the rate of 10% simple interest per annum realises later on that he could have taken as well from the co-operative society a loan at the rate of 8% simple interest per annum. He finds that for the interest that he has to pay after 3 years for the loan from the bank, he could have received Rs. 3,000 more as loan from the co-operative society. How much did he borrow from the bank?

27. Jayanta and Ajit take different sums as loan from the bank on the same day at the rate of 10% simple interest per annum. The amount which Jayanta pays after 2 years is equal to the amount Ajit will have to pay after 3 years. What is the ratio of their loans?

28. A person having taken a sum from a bank as loan finds after 5 years that he has to pay $\frac{2}{5}$th of the principal as interest. What is the rate p.c. per annum charged by the bank as simple interest? In how many years will a principal double itself at this rate of interest?

29. Mr. Rahaman takes a loan of Rs. 24,000 at the rate of 12% simple interest per annum from a bank to build a house. Just after one year he lets out the house at a rent of Rs. 520 per month. In how many years will he be able to pay off the loan with interest from the income of house rent?

30. An employee deposited with a bankhe sum he received as provident fund at the time of his retirement at the rate of 7% simple interest per annum. Every year he used to withdraw the interest and in 6 years he had drawn Rs. 1,134; what sum did he deposit? If he had not withdrawn the interest every year, in how many years would it amount to Rs. 4,779?

31. A farmer deposited a sum with the post office. After 4 years he came to know that it amounted to Rs. 434. He found that his interest was $\frac{6}{25}$th of his principal. What sum did he deposit and what was the rate of simple interest per annum in the post office?

32. A co-operative bank gives house-building loan to

low-income-group people on the condition that if the loan is paid back in full within 3 years it will charge 6% simple interest per annum, but if it exceeds 3 years the interest will be 7·5%. Kamalbabu and Bimalbabu borrow equal sums on the same date. Kamalbabu repays the loan in 3 years but Bimalbabu takes 5 years to do so. For this Bimalbabu has to pay an excess amount of Rs. 2,437·50 as interest. What sum did each of them borrow?

33. A person decided to make two time-deposits with a bank for his two daughters in such a manner that when each of his daughters would attain 18 years of age, she would receive Rs. 12,000 from her respective account. If the bank allows 10% simple interest per annum on time deposits and the present ages of his daughters are 13 years and 8 years respectively, find the sums he should deposit in respective accounts.

34. A sum deposited with a bank amounts to Rs. 1,107·60 in 7 years and Rs. 1,248 in 10 years. What was the sum and what was the rate per cent of simple interest per annum in that bank? If the bank allows 7·5% simple interest per annum, in how many years will that sum amount to Rs. 1,423·50?

35. A central co-operative bank gives long term loan at the rate of 4% simple interest per annum and village co-operative bank gives short term loan at the rate of 7·5% simple interest per annum. Five unemployed youths of the village Niamatpur take a long term loan of Rs. 43,750 and a short term loan of Rs. 20,000 and start a brick-field. After 3 years payment was made in full settlement of the short term loan. The amount of payment is found to be equal to the interest they pay later while settling the long term loan. After how many years do they pay off the long term loan?

36. A bank allows 10% and 11% simple interest per annum on time-deposits of 5 years and 7 years respectively. A person deposits a sum with the bank as 5 years time-deposit and on the same day, another sum, double the former, as 7 years-time-deposit. But due to some pressing needs he withdraws both the amounts after 5 years and as such incurs a loss of Rs. 1,890. How much did he deposit in respective accounts? How much did he receive as interest after 5 years?

37. A weavers' co-operative takes a long term loan for 10

years to purchase a power-loom and a short term loan for 2 years to meet the expenses of buying raw materials and other requisites. If the rates of simple interest per annum be 7% and 9% for long and short term loans respectively and the ratio of the sums of loans respectively and the ratio of the sums of loans 11 : 2, and, after two years they pay Rs. 3,245 in full settlement of the short term loan, find the sums of the two loans. How much will they have to pay for the long term loan?

38. A house building co-operative society decided to build 8 flats in a vacant land. For this, it borrowed Rs. 3,50,000 at a rate of simple interest of 12% per annum. After the completion of flats within two years, eight new members were taken on receiving Rs. 55,000 on the average from each of them and the society pays back to the bank the amount with interest and deposits the rest in the reserve fund of the society. What is the amount deposited in the reserve fund?

CHAPTER 5

Profit and Loss

In the prevailing economic system, there is hardly any iota of distribution of commodities during the course of transaction directly between the producer and consumer. On most of the occasions we find a middleman who makes the product available to the consumers by collecting it from the producers. In general, they do two types of work—wholesale and retail business. The wholesellers purchase a large share of the products from the producers and sell them to retail businessmen; retail sellers, in turn, sell commodities to consumers.

Most of them are self-employed enterpreneurs or enterprisers and they earn from the transaction. Generally speaking, their income occurs when they can sell out commodities at a price higher than the price by which they have purchased them. The excess money thus obtained is called 'profit'. But there are situations under which a dealer may be forced to sell his commodities at a price lower than the ammount by which he has purchased. In such cases, we say that the dealer has incurred 'loss'.

In class VI, you have worked out some problems of profit and loss and at that time you have come to know

Profit = selling price − cost price

and loss = cost price − selling price

In that class, you have come to know how a trader makes profit or incur loss, on the whole.

Remarks: As in the case of profit, selling price is greater than cost price and in the case of loss, cost price is greater than selling price, we can use the term 'loss' as negative profit, in the words of algebra.

Now let us consider another point :

Let us suppose that a bicycle - dealar purchases a bicycle for Rs. 500 and sell, it for Rs. 550. Again a radio mechanic spends Rs. 200 to produce a radio and then sells it at Rs. 250. You see that in both the cases the amounts of profit are the same. Rs. 50 is the amount of profit for both of them. On scrutiny you will see that the first man had invested Rs. 500 so as to get a profit of Rs. 50, whereas the second man invested only Rs. 200 to earn the same amount of profit, i.e. Rs. 50. You can easily understand that if the person who has invested Rs. 500, he would have earned a profit of Rs. 125 at the same rate. So from the standpoint of business, we cannot say that both are at the same level.

Profit and loss are related to business. That is why they should be compared in the context of capital invested.

In the meantime, you have learnt to make use of ratio, proportion, percentage etc. for comparison of inter-related objects. Here in this chapter you will learn how these can be used in problems related to profit and loss.

For the first case,

$$\begin{aligned}\text{Profit} : \text{Capital} &= 50 : 500\\ &= 1 : 10\\ &= \frac{1}{10}\end{aligned}$$

For the second case,

$$\begin{aligned}\text{Profit} : \text{Capital} &= 50 : 200\\ &= 1 : 4\\ &= \frac{1}{4}\end{aligned}$$

As $\frac{1}{4}$ is greater than $\frac{1}{10}$ therefore, in the second case the rate of profit is comparatively greater than that in the first.

If it is reduced into percentage., you will find.

Rate of profit of the first man is $(\frac{1}{10} \times 100)\% = 10\%$

and that of the second man is $(\frac{1}{4} \times 100)\% = 25\%$

Here also you see that profit per cent of the second man is higher than that of the first man.

Thus we see that if we can express profit and loss in percentage, then we can have a clear idea.

The aspects in connection with problems of profit and loss are :

(*i*) Cost price = The price at which a commodity is purchased.

N.B. : Sometimes cost price equals the capital of a purchaser. Again to producer the cost of production is the cost price.

(*ii*) Selling price = The price at which a commodity is sold.

(*iii*) Profit = Selling price − cost price.

(*iv*) Loss = Cost price − selling price.

(*v*) Profit or loss per cent = The profit of loss incurred by selling a commodity whose cost price is Rs. 100.

For example, if a commodity is purchased at Rs. 100 and sold for Rs. 110, then the profit will be $(110-100)\% = 10\%$. Again if a commodity is purchased at Rs. 100 and sold for Rs. 93, the loss will be $(100-93)\% = 7\%$.

N.B.: If nothing is stated to the contrary profit and loss per cent are calculated on cost price only.

(*vi*) Profit and loss can also be calculated in another way instead of rate per cent.

For instance, rate of profit or loss per Re. 1, rate of profit or loss per Rs. 10 etc. i.e. 10% profit or loss means profit or loss of 10 units on 100 units.

$\therefore$ Profit or loss on 1 unit will be $\frac{10}{100} = \frac{1}{10} = 0{\cdot}1$

Similarly, profit or loss on 10 units will be $\frac{1}{10} \times 10 = 1$

Determination of selling price :

Example 1. A book seller purchased a book at Rs. 12, and sold it at a profit of 15%, what will be the selling price of the book?

In the language of mathematics, the problem is

Cost price (in rupees)	Selling price (in rupees)
100	115 (∵ profit is 15%)
12	?

Now taking Rs. x for the required selling price and applying rule of three we have

$$x = 115 \times \frac{12}{100} = 13{\cdot}8$$

∴ The required selling price is Rs. 13·80.

Alternative method : Here, on finding the profit when it was sold at a profit of 15% and adding this to the cost price, the required selling price can be found out.

Then the problem in language of mathematics is

Cost price (in rupees)	Profit (in rupees)
100	15
12	?

Taking Rs. x for the required profit, we have

$$x = 15 \times \frac{12}{100} = 1{\cdot}8$$

That is the required total profit is Rs. 1·80 (*i*)

∴ The required selling price is Rs. (12 + 1·80) = Rs. 13·80 (*ii*)

Now let us analyse the above two results.

In (*i*) Total profit (in rupees) $= 15 \times \frac{12}{100} = 12 \times \frac{15}{100}$

= cost Price × profit per Rupee

In (*ii*) Selling price (in rupees) = 12 + 1·80 = cost price + Total profit = cost price + (cost price × profit per rupee)

In the case of loss, the above results take the form

(*i*) Total loss = cost price × loss per rupee

(*ii*) Selling price = cost price − total loss = cost Price − (cost price × loss per Rupee)

Now we can obtain algebraic formula from the above results. Let us suppose,

Cost price or production cost = C
Selling price = S
Profit per rupee = p
Lost per rupee = l (loss p.c./100)
Total profit = P
Total loss = L

Then in the case of profit

$$P = C \times p = C.p$$
$$S = C + C.p$$
$$= C(1+p)$$

Corrollary : $C = \dfrac{S}{1+p}$

In the case of loss

$$L = C \times l = C.l$$
$$S = C - C.l$$
$$= C(1-l)$$
$$C = \frac{S}{1-l}$$

[Afterwards when you will be required to determine cost price, these formulae will be of great help.]

Solution using the algebraic formula :

Example 2. A trader sells his commodities at 12% profit. What will be the selling price of the commodities which he has purchased for Rs. 240?

In the language of mathematics, the problem is

Cost price (in rupees)	Selling price (in rupees)
100	112 (∵ Profit is 12%)
240	?

Here $C = 240, p = \frac{12}{100} = 0{\cdot}12.\ S = ?$

According to the formula, $S = C(1+p)$

$$= 240\,(1+0{\cdot}12)$$
$$= 268{\cdot}8$$

$\therefore$ The required selling price is Rs. 268.80

Determination of cost price or cost of production :

Example 3. A weaver sells a 'Shari' for Rs. 140 and makes a profit of 25%. What is the production cost of each Shari?

In the language of mathematics, the problem is

Production cost (in rupeees)	Selling price (in rupees)	
100	125	($\because$ profit is 25%)
?	140	

Taking Rs. x for the required production cost we get,

$$x = 100 \times \frac{140}{125} = 112$$

$\therefore$ Required production cost = Rs. 112

Now, let us find the production cost with the help of the corollary.

Here $S = 140, p = \frac{25}{100} = {\cdot}25,\ C = ?$

According to the formula, $C = \frac{S}{(1+p)}$

$$= \frac{140}{1+0{\cdot}25} = 112$$

That is, the required production cost is Rs. 112.

Determination of profit or loss per cent :

Example 4. A shop-keeper purchases 35 kg of 'Dal' at Rs. 5 per kg and sells the whole of it for Rs. 210. Calculate his profit or loss per cent.

The cost price of 35 kg of 'Dal' at Rs. 5 per kg $= \text{Rs. } 5 \times 35 = \text{Rs. } 175$..

By inspection we see that the selling price is more than the cost price, so the shop-keeper will earn profit.

Now in the language of mathematics, the problem is

Cost price (in rupees)	Selling price (in rupees)
100	?
175	210

Taking Rs. x for the selling price in the first row, we have

$$x = 210 \times \frac{100}{175} = 120$$

∴ the required profit per cent will be $(120 \times 100) = 20$ or rate or profit is 20%.

[Remark : The problem could have been solved also without finding selling price from the following table :

Cost price (in rupees)	Profit (in rupees)
100	?
175	35]

Application of the algebraic formula :

Here, $C = 175$, $S = 210$, $p = ?$

We know, $S = C(1 + p)$

i.e. $210 = 175(1 + p)$

$$\text{or, } 175p = 210 - 175 = 35 = \frac{35}{175} = 0{\cdot}2$$

That is, profit per rupee is Rs. 0·20

∴ the profit p.c. is $0{\cdot}20 \times 100 = 20$

or the profit is 20%

Example 5. Haripada Basak sold a stove for Rs. 423 and incurred a loss of 6%. At what price should it be sold so as to earn a profit of 8%?

In the language of mathematics, the problem is

In case of 6% loss		In case of 8% profit	
Cost price (in rupees)	Selling price (in rupees)	Cost price (in rupees)	Selling price (in rupees)
100	94	100	108
?	423	?	?

From the above two tables it is clear that by solving the first one we will have the cost price of the stove. Then if we put the cost price in the second table and solve it, the required price will be obtained.

Taking Rs. x for the cost price, from the first table we have

$$x = 100 \times \frac{423}{94} = 450$$

$\therefore$ the cost price of the stove is Rs. 450.

Now posting this cost price in the second table and taking Rs. y for the required selling price, we have

$$y = 108 \times \frac{450}{100} = 486$$

$\therefore$ the required selling price is Rs. 486.

Application of the algebraic formula :

In the first part, $C = ?\ S_1 = 423,\ l = 0{\cdot}06$

So we have $C = \dfrac{423}{1-0{\cdot}06} = 450$

Now in the second part, we have

$C = 450,\ S_2 = ?,\ p = 0{\cdot}08$

$\therefore\ S_2 = C(1+p) = 450(1+0{\cdot}08)$

$= 450 + 36 = 486$

$\therefore$ the required selling price is Rs. 486.

[*Remark :* The result could have been obtained from the selling price columns of the two tables, as the cost price columns will be same.

Taking Rs. y for the required selling price we get

$$y = 108 \times \frac{423}{94} = 486$$

$\therefore$ the required selling price = Rs. 486.]

Example 6. Two retail dealers purchase radios from the same wholesale dealer at Rs. 400 each. One of them sells at 20% profit; the other one marks the selling price by increasing 30%, but allows 8% rebate to the consumers. How much do each of them gain by selling a radio? Who makes the greater profit?

As the first retailer makes 20% profit,
So his selling price per radio = Rs. $400\ (1+0{\cdot}2)$
= Rs. 480.

As the second retailer fixes price increasing 30% of the cost price,
So the marked price per radio = Rs. $400\ (1+0{\cdot}3)$
= Rs. 520.
But as he allows 8% rebate on the marked price,
So his selling price per radio = Rs. $520\ (1-0{\cdot}08)$
= Rs. $478{\cdot}40$
$\therefore$ The first retailer's profit = Rs. $(480-400)$ = Rs. 80.
and the second retailer's profit = Rs. $(478{\cdot}40-400)$ = Rs. $78{\cdot}40$.

$\therefore$ The first retailer makes a profit of Rs. $(80-78{\cdot}40)$ = Rs. $1{\cdot}60$ more than the second retailer.

[*N.B.:* Rebate and discount are generally calculated on marked price.]

Example 7. By selling an article a trader incurred a loss of 20%. Had he been able to sell it for Rs. 100 more he would have made a profit of 5%, what is the cost price of the article?

Let the cost price be Rs. x, the selling price at a loss of 20% be Rs. S_1 and at a profit of 5% be Rs. S_2.

$\therefore\ S_1 = x(1-0{\cdot}2) = 0{\cdot}8x$

and $S_2 = x(1+0{\cdot}05) = 1{\cdot}05x$.

Now as per condition of the problem,

$S_2 = S_1 + 100$

that is, $1{\cdot}05x = 0{\cdot}8x + 100$

$$\text{or, } 1{\cdot}05x - 0{\cdot}8x = 100$$
$$\text{or, } {\cdot}25x = 100$$
$$\therefore\ x = \frac{100}{{\cdot}25} = 400$$

$\therefore$ the required cost price of the article is Rs. 400.

Alternative method :

Since there is a profit of 5% if the selling price be Rs. 100 more, this Rs. 100 includes the loss of 20% of the cost price and profit of 5%.

$\therefore$ (20+5)% i.e. 25% of cost price is Rs. 100.

$\therefore$ the cost price is Rs. $100 \times \frac{100}{25}$ = Rs. 400.

Example 8. In a shoe-trade, each of the producer, the wholeseller and the retailer make a profit of 20%. What is the cost of production of a pair of shoes for which a customer pays Rs. 86·40? Compare the amounts of profit of the retailer and the wholeseller. If the customer could purchase through a state trading agency at the selling price of the producer, how much would he have saved?

We know, consumer's cost price = Rs. 86·40 = retailer's selling price.

$\therefore$ the retailer's cost price = Rs. $\frac{86{\cdot}40}{1+0{\cdot}2}$ = Rs. 72.

Again, retailer's cost price = wholeseller's selling price.

$\therefore$ the wholeseller's cost price = Rs. $\frac{72}{1+0{\cdot}2}$ = Rs. 60.

Once again, wholeseller's cost price = producer's selling price.

$\therefore$ Producer's cost of production = Rs. $\frac{60}{1+0{\cdot}2}$

= Rs. 50.

Thus (*i*) The cost of production of the particular pair of shoes = Rs. 50.

(*ii*) The retailer's profit is Rs. (86·40−72) = Rs.

14·40, and the producer's profit is Rs. (60−50) = Rs. 10.

∴ The retailer makes a profit of Rs. (14·40−10) = Rs. 4·40 more than the producer.

(*iii*) If the consumer could purchase a pair of shoes at a price equal to the producer's selling price, then he had to pay Rs. 60.

∴ he would have saved Rs. (86·40−60) = Rs. 26·40

Example 9. A tea merchant prepares a special blend by mixing Darjeeling tea of Rs. 50 per kg and Assam tea of Rs. 35 per kg in the ratio 2 : 5. If he sells this quality of blend tea at Rs. 44 per kg, then what will be his profit or loss per cent?

Let us suppose, he mixes 2 kg of Darjeeling tea with 5 kg of Assam tea.

So the cost price of the new quality of tea per kg will be

$$= \text{Rs. } \frac{(2\times 50)+(5\times 35)}{7} = \text{Rs. } \frac{275}{7}$$

In the language of mathematics, the problem stands as

Cost price (in rupees)	Selling price (in rupees)
$\frac{275}{7}$	44
100	?

Taking Rs. x for the required selling price, by the rule of three we have,

$$x = 44 \times \frac{100}{\frac{275}{7}} = \frac{44\times 100\times 7}{275} = 112$$

∴ his rate of profit will be (112−100)% = 12%.

Example 10 : A hawker purchases some mangoes at the rate of 9 mangoes for Rs. 4 and sells them at Rs. 7 per dozen. What will be his profit or loss p.c.?

9 mangoes cost Rs. 4.

$\therefore$ 1 mangoe costs Re. $\frac{4}{9}$

Again, 12 mangoes are sold at Rs. 7

$\therefore$ 1 mangoe is sold at Re. $\frac{7}{12}$

Now in the language of mathematics, the problem is

Cost price (Rs.)	Selling price (Rs.)
$\frac{4}{9}$	$\frac{7}{12}$
100	?

Now taking Rs. x for the required selling price, by the rule of three we have

$$x = \frac{7}{12} \times \frac{100}{\frac{4}{9}} = \frac{7 \times 100 \times 9}{12 \times 4} = 131{\cdot}25$$

In other words the hawker sells at Rs. 131·25.

$\therefore$ rate of profit = $(131{\cdot}25 - 100)\% = 31{\cdot}25\%$.

Example 11 : A consumer's co-operative society purchases each piece of 'Shari' from a wholesale dealer for Rs. 75 and sells it to its members at 5% profit, while selling Sharis to the society the wholesale dealer, of course, charges 25% profit on the price at which he purchases it from weaver.

(*i*) How much does a member of the society pay for a 'Shari'?

(*ii*) For what price does the wholesale dealer purchase a 'Shari' from the weavers?

(*iii*) If the society could have acquired 'Shari' directly from the weaver at the price which the wholesale dealer pays, how much would each member have to pay for a 'Shari'?

(*iv*) Calculate how much would the members have saved for each piece of 'Shari'.

(*i*) As the society makes 5% profit, So the cost price of a 'Shari' to its members or the selling price of the society for each piece is Rs. 75(1+0·05) = Rs. 78·75.

(*ii*) As the wholesale dealer sells a 'Shari' for Rs. 75 at 25% profit, so his cost price is $\frac{\text{Rs. } 75}{1+0{\cdot}25}$

= Rs. 60.

(*iii*) If the society could purchase a 'Shari' for Rs. 60, then at 5% profit its selling price would have been Rs. 60(1+0·05) = Rs. 63 = Cost price of its members.

(*iv*) In that case a member would have saved Rs. (78·75−63) = Rs. 15·75 for each 'Shari'.

Example 12. Last year the peasants of kumirmari village have formed a co-operative grain godown. So the peasants are not now compelled to sell paddy at a lower rate at the time of harvest. Mohan Biswas a peasant of that village has invested Rs. 2700 on seed, manure, wages etc. and has produced 19 quintals of paddy. At the time of harvest, paddy is being sold at Rs. 180 per quintal. But Sree Biswas does not sell it in the market, instead he deposits the lot in the co-operative godown and takes a loan of Rs. 2500. After six months he sells the whole lot at current market price and after paying off the loan of the society with 5% interest, he receives a cash of Rs. 1332·50 in hand,

(*i*) At what rate per quintal did he sell the paddy?

(*ii*) If he had sold the lot at the time of harvest, what would have been the amount of his loss?

(*i*) After six months the amount he has to pay to the co-operative in full settlement of the loan = Rs. 2500 $(1+\frac{1}{2}\times 0{\cdot}05)$ = Rs. 2562·50 .

Again he had in cash Rs. 1332·50.

So the sale-proceeds of 19 quintals
= Rs. (2562·50+1332·50) = Rs. 3895.
∴ rate of sale-proceeds per quintal

$$= \text{Rs. } \frac{3895}{19} = \text{Rs. } 205.$$

(*ii*) If he had sold the whole lot at the time of harvest, the total sale-proceeds would be Rs. 180×19=Rs. 3420·00.

But as he sells it after six months he receives a total sum of Rs. (2500+1332·50) = Rs. 3832·50.
∴ his loss would have been Rs. (3832·50−3420)
= Rs. 412·50.

Exercise 10

1. A watch dealer purchases a watch for Rs. 240. For how much should he sell it to get a profit of $17\frac{1}{2}\%$?

2. A fruit seller purchases a basketful of mangoes and found that some mangoes are rotten. Now by selling the rest for Rs. 22·80 he incurred a loss of 5%. How much did the mangoes cost him?

3. A hawker purchased two dozen frocks for Rs. 192 and one dozen for Rs. 84. By selling the whole lot he made a profit of 25%. Find how much did he receive by selling the whole lot.

4. A fish dealer purchased 35 kg of fish for Rs. 525 and sold the whole lot at Rs. 18 per kg. What will be his gain or loss per cent?

5. The cost of production of an wooden almirah including wages is Rs. 837. For how much should it be sold to get a profit of 20%?

6. One of the two hawker friends purchases some clothing for Rs. 300 and sells them for Rs. 350; while the other purchases some baloons for Rs. 25 and sells them for Rs. 30. Compare their percentages of profit.

7. A tea merchant makes a special blend of tea by mixing 2 kg of Darjeeling tea at Rs. 45 per kg with 7 kg of Assam tea at Rs. 25 per kg and sells the new quality of tea at Rs. 31·80 per kg. Calculate his gain or loss per cent.

8. Charging 30% above its production cost a radio maker puts a label of Rs. 286 on a radio as its price. But at the time of selling it, he allows 10% discount on the labelled price. What will be his gain per cent?

9. A retail dealer in medicine gets 20% commission on marked price; but he sells them to his customers at marked price. Calculate his gain per cent.

10. A paper dealer purchases 18 reams of paper at Rs. 125 per ream. He sells 10 reams at Rs. 140·50 per ream and 8 reams at Rs. 145 per ream. Calculate his gain per cent.

11. A book seller imports foreign books, worth Rs. 10,000 and pays $12\frac{1}{2}$% import duty on that value. For what amount should he sell the whole lot to gain 15% over his investment?

12. A radio dealer sells a radio at a loss of 3% for Rs. 363·75. For how much should he have sold it to gain 10%?

13. A bicycle dealer sells a bicycle for Rs. 840 and thereby gains 12%. What would have been his gain p.c. if he had sold it for Rs. 26·25 more?

14. A weavers' co-operative sells 'shari' to cloth merchants at a profit of 25%. The merchant in his turn sells it to his customers for Rs. 75. If the cost of production of the 'shari' is Rs. 50, what per cent of profit does the merchant get?

15. A contractor received two orders. For one he will have to spend Rs. 25,000 and will receive Rs. 30,500 against the bill. For the other he is allowed to charge 20% on investment. Which one of the two is more profitable to him?

16. A cabinet maker is ready to supply benches at Rs. 225 per bench. But the school authority spent Rs. 4,800 on wood and Rs. 137·50 on screw etc. and Rs. 2,150 as wages to the carpenters to make 35 benches for the students. Calculate the per centage of saving the school makes by this.

17. In what ratio Darjeeling tea at Rs. 32 per kg be mixed with Assam tea at Rs. 25 per kg to have a quality of tea which when sold at Rs. 32·40 would bring a profit of 20%?

18. A contractor has received an order amounting to

Rs. 15,250 for the construction of an wooden bridge. How much should he spend so that he may get a profit of 22%?

19. A wireless mechanic produced a radio set and a tape-recorder and sold them at 15% and 20% profit respectively. His total profit was Rs. 262·50. If the ratio of the costs of production of the two be 1 : 3, find the costs of production of the radio set and the tape-recorder.

20. The marked price of a fountain pen of a particular type is Rs. 36·50. A shop-keeper sells one pen to a customer at a discount of Rs. 2·90 and gains 12%. If he sells the second one to another customer for Rs. 34·50, what will be his gain per cent?

21. The cost of production of a hurricane lantern is Rs. 28. The producer sells it to the shop-keeper at 22% profit, what will be the gain p.c. of the shop-keeper if the sells one for Rs. 42·70?

22. A craftsman purchases wood and steel worth Rs. 26·50 and produces two ploughs working two days on them. He sells them for Rs. 35·50 respectively. If his wage is calculated at Rs. 18 per day, what will be his gain of loss p.c.?

23. A village hawker procures eggs from peasant- households at Rs. 7·80 per dozen and sells them to the wholesale dealer at Rs. 79·30 per hundred. What is his gain p.c.?

24. A fruit dealer purchases one gross of banana at Rs. 3 per dozen and sells 8 dozen at Rs. 3·75 per dozen. For how much should he sell the remaining banana to have an overall profit of 20%? [1 gross = 12 dozen]

25. A person takes a loan from a bank at the rate of 12% simple interest per annum to purchase two houses. After two years he sells one for Rs. 56,200 and the other for Rs. 51,800. In this transaction he gains 20% on the cost price of the houses after paying back loan with interest. What was the sum of the loan? If the cost prices of the houses are in the ratio 11 : 9, what are the amounts of his gain from respective houses?

26. A contractor hoping to construct a road for Rs. 25,000, quotes a price 29·6% higher. But due to sudden rains the cost rises by 8%. What will be his gain per cent?

27. To prepare 2,000 copies of a book a publisher spends

Rs. 3,875 on paper, Rs. 3,315 on printing and Rs. 810 on binding. Now after allowing 20% commission to the book sellers, the publisher wants to retain 20% profit, what should be the marked price of each book?

28. The cost of production and the cost price at different stages of a bicycle are :

Cost of production Rs. 350	Wholesale cost price Rs. 420
Retail cost price Rs. 483	Consumer's cost price Rs. 555·45

(*i*) What is the gain per cent of the retailer?
(*ii*) What is the gain per cent of the producer?
(*iii*) What is the gain per cent of the wholeseller?
(*iv*) What per cent in excess of the cost of production does a consumer pay for a bicycle?
(*v*) If the consumer could procure a bi-cycle direct from the producer allowing him 30% profit, how much could he save?

29. Here is some data regarding the production and expenses of a weavers' co-operative society :

Quantity produced	Cost of yarn	Wages	House rent, depreciation and other expenses
4900 metres	Rs. 29400	Rs. 17150	Rs. 12250

Profit earned by the society = 25%, by the agents = 20% and by the retailers = 15%.

(*i*) What is the rate per metre does the society charge the agents?
(*ii*) At what rate per metre does the retailer purchase?
(*iii*) How much per metre does the consumer pay?
(*iv*) What is the difference between the cost of production and the cost price of the consumer per metre?
(*v*) If the soceity opened sales counter of its own and charged 30% profit, how much did the consumers save per metre?

30. (*i*) In the cost of production of a particular spare part in a factory the elements – raw-material, wages and other expenses are in the ratio 5 : 3 : 2. If by selling a spare part for Rs. 988 the

factory gains 30%, find the expenses for each element in the cost of production.

(*ii*) If due to rise in price level cost of raw-material, wages and other expenses rise by 25%, 10% and 20% respectively, then by what per cent the selling price of the spare part will be raised to keep the rate of profit of the factory undisturbed?

31. A wholesale buyer purchases from a village a buffalo and an ox at Rs. 3000. He sells the buffalo at 15% gain and the ox at 10% loss, but overall gain is $8\frac{1}{3}$%. Find the cost price of the buffalo and the ox.

32. An office co-operative store purchases 2 kg tins of mustard oil from the wholesale market and sells to its members for Rs. 30·45 at a profit of 5%. But if they could purchase the same directly from oil mill and charged Rs. 26·75 per tin they could earn a profit of 7%, what profit p.c. does the wholeseller charge?

33. After production a cabinet-maker finds that the ratio of the cost of production of a dressing-table and an almirah is 1 : 3. He sells the almirah and the dressing-table at $22\frac{1}{2}$% and 20% profit respectively and earns a total profit of Rs. 332·50. Find the cost of production of the respective items.

34. In the market it is a general practice that the producer, the wholeseller and the retailer earn 15%, 20% and 25% profit respectively. Now if a commodity reaches the hands of a consumer from a producer through the wholeseller and the retailer in due course, then,

(*i*) What is cost of production of a commodity for which the consumer pays Rs. 13·80?

(*ii*) how much will the consumer pay for a commodity, the cost of production of which is Rs. 14?

(*iii*) how much does a consumer pay to get a commodity for which the retailer pays Rs. 9·80?

(*iv*) how much does a consumer pay for a commodity for which the wholeseller pays Rs. 17·50?

(*v*) when the consumer who purchases a commodity for Rs. 27·60, could purchase it directly from the producer allowing 35% profit, what would have his savings?

35. A broker purchased two old houses for Rs. 39,100 each.

The first he sold at 15% profit and the second at a loss of 7·5%. What was overall gain or loss?

36. A dealer purchased two tables for Rs. 800. One he sold at 10% profit and the other at 20% profit. But if he could gain 20% on the first and 15% on the second, the total sale-proceeds would have been Rs. 6·50 more. Find the cost price of the respective table.

Arithmetic

Exercise 1

1. (i) 3 : 2 (ii) 5 : 6 (iii) 5 : 13 (iv) 2 : 1 (v) 17 : 20 (vi) 4 : 5 (vii) 5 : 3 2. 1 : 8 3. 29 : 15 4. (i) 13 : 17 (ii) 45 : 8 5. (i) 5 : 8 (ii) 12 : 7 (iii) 13 : 22 (iv) 21 : 25 (v) 83 : 48 (vi) 12 : 125 (vii) 118 : 7 (viii) 225 : 129 6. (i) 4 : 3 (ii) 1 : 2 (iii) 2 : 1 (iv) 5 : 8 (v) 5 : 7 (vi) 37 : 9 (vii) 8 : 9 (viii) 7 : 10 (ix) 4 : 15 (x) 7 : 15 (xi) 7 : 15 (xii) 10 : 17 (xiii) 9 : 10 7. 35, 60, $\frac{1}{2}$, 6·5, 0·96, 3·8, 1, q^2 8. 3 : 4 : 10 9. 3 : 2 : 4 : 5 10. (i) 3 : 2 (ii) 8 : 11 (iii) 11 : 9 11. 20 yrs. 12. 42 13. Rs. 8 14. 13m 2dcm 15. 72 km 16. red : white = 25 : 24 17. 1 : 1 18. 3 : 2 19. 4 : 3 and 4 : 1 20. (i) 15 : 8 : 14 (ii) 19 : 10 : 17 (iii) 23 : 10 : 19

Exercise 2

1. (i) 8 : 10 = 16 : 20, 10 : 8 = 20 : 16, 10 : 20 = 8 : 16, 20 : 10 = 16 : 8 (ii) 30 : 24 = 5 : 4, 24 : 30 = 4 : 5, 5 : 30 = 4 : 24, 30 : 5 = 24 : 4 (iii) not proportional (iv) 4 : 10 = 12 : 30, 10 : 4 = 30 : 12, 4 : 12 = 10 : 30, 12 : 4 = 30 : 10 (v) $\frac{1}{5} : \frac{1}{10}$, = $\frac{1}{16} : \frac{1}{32}$, $\frac{1}{10} : \frac{1}{5} = \frac{1}{32} : \frac{1}{16}$, $\frac{1}{10} : \frac{1}{32} = \frac{1}{5} : \frac{1}{16}$, $\frac{1}{32} : \frac{1}{10} = \frac{1}{16} : \frac{1}{5}$

2. (i) 16 (ii) 24 (iii) $\frac{1}{18}$ (iv) 9 (v) 12 (vi) 0·04 (vii) 0·8 3. (i) 15 (ii) 8 (iii) 0·55 (iv) $\frac{1}{2}$ 4. (i) 4 : 8 = 8 : 16 (ii) 9 : 12 = 12 : 16 (iii) $5\frac{1}{4} : 7 = 7 : 9\frac{1}{3}$ (vi) $\frac{2}{3} : \frac{4}{9} = \frac{4}{9} : \frac{8}{27}$ (v) 1·6 : 2·8 = 2·8 : 4·9 5. (i) 5 : 15 =15 : 45 (ii) 25 : 45 = 45 : 81 (iii) $\frac{4}{9} : \frac{10}{21} = \frac{10}{21} : \frac{25}{49}$ (iv) $\frac{3}{7} : \frac{4}{7} = \frac{4}{7} : \frac{16}{21}$ (v) 1·5 : 4·5 = 4·5 : 13·5 (vi) $1\frac{4}{7} : 3\frac{1}{7} = 3\frac{1}{7} : 6\frac{2}{7}$ 6. (i) proportional (ii) proportional (iii) proportional 7. 30

8. $\frac{9}{16}$ 9. 8 : 12 : 15 10. 6·72gm 11. (i) 50 (ii) 55 12 (i) 10·5kg and 15·5kg (ii) Rs. 252 13. ammonia 16 quintals and potash 24 quintals 14. 468

Exercise 3

1. Rs. 120 **2.** 18 days **3.** 200 m **4.** 12 days **5.** 5600 **6.** 1·320 kg **7.** 2 hours 20 minutes **8.** 200 days **9.** 28 days **10.** 1500 km **11.** 24 days **12.** 46 bundles **13.** 90 **14.** 24 kahans **15.** $34\frac{3}{8}$ quintals **16.** 14 days **17.** 24 bighas **18.** 5 days **19.** 9m **20.** 720 gms **21.** 50 **22.** 70 pages **23.** Rs. 13.50 **24.** 10 25. 25 days **26.** 16 **27.** 10 hours **28.** 15

Exercise 4

1. 531, 2124 2. organic 144 bags, chemical 32 bags, 3. 5 quintals 4. Ram Rs. 90, Shyam Rs. 210, Jadu Rs. 420 5. Biltu Rs. 176·25, Mintu Rs. 352·50, Aloke Rs. 411·25 6. 1st Rs. 26,500, 2nd Rs. 42,400, 3rd Rs. 15,900 7. Oxygen 177·8 kg, Hydrogen 22·2 kg 8. literate 20·46 crores, Illiterate 41·54 crores 9. 2,555 10. family expense Rs. 1200, education Rs. 400 11. Rs. 25,506 12. A Rs. 160, B Rs. 240, C Rs. 300 13. 1st brother : 2nd brother : sister = 133 : 82 : 40 14. Rs. 60,000, Rs. 12000, Rs. 8000 15. 7 : 1 16. 5 : 4, 15 : 13, 335 : 278 17. Rs. 7,500 18. 28 : 5 19. Rs. 2,800 20. (i) 12,348 (ii) 25 : 16 21. 25 : 21 22. 44 crores, 11 crores more.

Exercise 5

1. Nitai Rs. 630, Jabbar Rs. 1,050 **2.** Abed Rs. 1,500, Uttam Rs. 1,800, Jafar Rs. 1,650 **3.** A– Rs. 150, B– Rs. 100, C– Rs. 175 **4.** Madhu Rs. 500 **5.** Rs. 60 **6.** Sukanta Rs. 4,700, Susanta Rs. 5,300, Sudhir Rs. 4,400 **7.** Tapas Rs. 1,120, Nihar Rs. 1,400, Biswajyoti Rs. 1,680 **8.** Keshab Rs.

18,450, Madhab Rs. 22,140, Jadab Rs. 20,910 **9.** 1st Rs. 900, 2nd Rs. 1,200, 3rd Rs. 750; 6 : 8 : 5 **10.** 1st Rs. 5,610, 2nd Rs. 4,818, 3rd Rs. 4,202 **11.** A- Rs. 6,478, B- Rs. 7,380 **12.** First three Rs. 1,643 each, next two Rs. 1,720·50 each. **13.** Mahendra Rs. 3,248, Mannan Rs. 3,024, Rupchand Rs. 3,472. **14.** Gobinda Rs. 6,960, Mukunda Rs. 8,700, Ananda Rs. 3,915 **15.** 1st Rs. 3,650, 2nd Rs. 3,500, 3rd Rs. 3,950. **16.** Amal Rs. 12,000, Anup Rs. 16,750, Anwar Rs. 7,250.

Exercise 6

1. 5 : 7 **2.** 2 : 1 **3.** 3·5kg **4.** 2 litres. **5.** 3·5 kg **6.** Lead 2·55 quintals. Antimony 0·45 quintals. Tin 0·15 qu. **7.** Ammonia - 88 kg, Phosphirus- 40 kg, Potash, 16 kg. **8.** 18 m, Red 20m Blue 10m **9.** 91 : 81 **10.** 6 : 1, 24 : 5 **11.** 0·5 kg. **12.** 3 litres **13.** 1·5 kg **14.** $\frac{1}{3}$ parts **15.** $\frac{4}{7}$ parts. **16.** 7·17 **17.** 31 : 17 **18.** 2 kg 19. Cowdung : Ammoni : Potash = 11 : 4 : 2 **20.** 5 : 2 **21.** 1 : 2 **22.** 15 : 8

Exercise 7

1. (i) $\frac{1}{4}$ (ii) $\frac{2}{5}$ (iii) $\frac{39}{50}$ (iv) $\frac{1}{6}$ (v) $\frac{5}{8}$ (vi) $\frac{1}{16}$ (vii) $\frac{1}{30}$ (viii) $\frac{1}{32}$ **2.** (i) 0·35 (ii) 0·28 (iii) 0·5 (iv) 0·07 (v) 0.3125 (vi) 0·012 (vii) 0·72 **3.** (i) 50% (ii) $37\frac{1}{2}$% (iii) $92\frac{1}{2}$% (iv) 320% (v) 705% (vi) 3% (vii) 70% (viii) 440% (ix) 75% **4.** (i) Rs. 10 (ii) 160 m. (iii) 26 kg (iv) Rs. 219 **5.** (i) 25% (ii) $3\frac{1}{4}$% (iii) 75% (iv) 3·125% **6.** 70%, 30% **7.** 66·6%, 33·3% **8.** 20%, 30%, 50% **9.** 1962 **10.** 15318 **11.** 798 **12.** 7786 bighas **13.** 7 qu. **14.** 198 kg **15.** 86·5% 16. 33% **17.** Rs. 1328·25 **18.** 89·25 km **19.** 18·9 litres, 5·76 liters, 0.34 litres **20.** Rs. 1,08,000, Rs. 27,000, Rs. 45,000, Rs. 72,000 **21.**

35% **22.** (i) Rs. 5125 (ii) Rs. 16500 (iii) 164% **23.** 11 : 18 **24.** 15360 **25.** 208·50 **26.** 20% **27.** Rs. 5250, Rs. 1312.50 **28.** (i) 585 (ii) 12 (iii) 42 (iv) 154 : 117 : 29 **29.** (i) 39 (ii) 12 (iii) 3 (iv) 6 **30.** 52% **31.** 8·5 kg, 1·5 kg, 625 gr **32.** 3 hour 32 min. **33.** Rs. 80.50 **34.** (i) 24%, 26%, (ii) 75·66 (approx.), 72·89 (approx.) (iii) 20% (iv) 36%, 40%, 0·8% (v) 16% (vi) 10%.

Exercise 8

1. (i) $3\frac{7}{8}$ (ii) $\frac{5}{8}$ (iii) 2 (iv) 1 (v) 5 (vi) 998·4 (vii) 1 (viii) 1

2. (i) $\frac{1}{13}$ (ii) 3 m (iii) $\frac{3}{20}$, 33 acres, (iv) For cultivation Rs. 17, 850, Each member Rs. 2,856 (v) Rs. 12,000 (vi) Illiterate $\frac{31}{40}$, 1,30,200 ; $\frac{9}{31}$ (vii) Road Rs. 6,720, School Rs. 4,480, others $\frac{1}{8}$ part.

Miscellaneous

1. $\frac{9}{16}$ **2.** 1·25 kg **3.** 24 yrs., 18 yrs. **4.** 26·6 quintals **5.** (i) (a) 3 : 4 (b) 3 : 4 (c) 10 : 11 (d) 4 : 5 (ii) 16 kg (iii) 30·56 kg (approx.) **6.** 11 : 6 **7.** Arun Rs. 12,500; Shymal Rs. 8,800; Rashid Rs. 10,200 **8.** One rupee - 78, fifty paise - 132 , 25 paise - 168 **9.** 1 : 7, 0·1 litre **10.** 12 H.P. **11.** 3 days **12.** 2·56 tonnes **13.** 9 hrs. **14.** 2352 sq.m **15.** 20000 times, 3.5 m **16.** (i) 0·6, 66·6, 16·6 (iii) 0·1, 10 (iv) $\frac{7}{200}$, 3·5 (v) $\frac{1}{25}$, 4 (vi) $\frac{1}{125}$, 0.0075. (viii) $\frac{1}{8}$, 0·125 (ix) $\frac{11}{100}$, 0·11 (x) $\frac{7}{40} = 0{\cdot}175$ **17.** (i) Rs. 3·50 (ii) 125 gm (iii) 6·75 m (iv) 3·6 km (v) 1·72 sq. cm. (vi) Rs. 1·27 (vii) 144 tonnes (viii) 0·01 **18.** (i) 17·5% (ii) 12·5% (iii) 37·5 (iv) 800 persns **19.** (i) 2·044 (approx.) (ii) 1975 (iii) 41·667, 2·778, 92 : 125 (approx.) (iv)

2:5, 17:23, 54·12 (v) 7·4% decrease (approx.) (vi) 1 : 2, 15 : 33, 113·333 (vii) 11·224% (approx.) more, 1·224% more **20.** 800, 544 **21.** 21·75 quintals, 5·25 quintals, 35% **22.** (i) 1050 Bighas (ii) 798 Bighas, 24% (iii) 13·1% (iv) 266 : 125 : 59 (v) 14% **23.** 196 persons **24.** (i) 625 persons (ii) 200 persons (iii) 340 persons **25.** (i) 16800 (ii) 252 (iii) 55·125% (iv) 3087 (v) 18·375% **26.** (i) Rs. 8100 (ii) 6 : 5 (iii) Rs. 30600 **27.** (i) Rs. 32000 (ii) Rs. 3600 (iii) Rs. 20,400 (iv) Rs. 8000.

Exercise 9

1. Rs. 1575, Rs. 5075 **2.** Rs. 87·50 **3.** Rs. 2500 **4.** 5 yrs. **5.** 525 **6.** 7 yrs. **7.** Rs. 3000 **8.** 5 yrs. **9.** 12·5 yrs. **10.** 12% **11.** 3 : 4 **12.** Rs. 60000 **13.** Rs. 10750, Rs. 5912·50 **14.** 10 : 9 **15.** Rs. 3500 **16.** 12·5% **17.** Rs. 300 **18.** Rs. 42.50 **19.** Rs. 8000 **20.** 9·5% **21.** Rs. 7000 **22.** Rs. 7900 **23.** Rs. 7115·50 **24.** Rs. 3200, Rs. 2500 **25.** 16 yrs. **26.** Rs. 12000 **27.** 13 : 12 **28.** 8%, 12·5 yrs. **29.** 9 yrs. after taking the loan **30.** Rs. 2700, 11yrs. **31.** 350, 6% **32.** Rs. 12500 **33.** Rs. 8000, Rs. 6000 **34.** 6%, Rs. 780, 11yrs. **35.** 14 yrs. **36.** Rs. 3500, Rs. 7000, Rs. 5250 **37.** Short term - Rs. 2750, Long term Rs. 15125, Rs. 25712·50 **38.** 6000

Exercise 10

1. Rs. 282 **2.** Rs. 24 **3.** Rs. 345 **4.** 20% gain **5.** Rs. 1004·40 **6.** 5 : 6 **7.** 8% gain **8.** 17% **9.** 25% **10.** 14% **11.** Rs. 12937·50 **12.** Rs. 412·50 **13.** 15 5% **14.** 20% **15.** First one **16.** 10% **17.** 2 : 5 **18.** Rs. 12500 **19.** Rs. 350, Rs. 1,050 **20.** 15% **21.** Rs. 8·54, 25% **22.** 14% gain **23.** 22% **24.** Rs. 3·30 **25.** Rs. 75000, Rs. 8250, Rs. 6750 **26.** 20% **27.** Rs. 6 **28.** (i) 15% (ii) 20% (iii) 15% (iv) 58·7% (v) Rs. 100·45 **29.** (i) Rs. 15 (ii) Rs. 18 (iii) Rs. 20·70 (iv) Rs. 8·70

(v) He would have received Rs. 5·10 less per metre **30.** (i) Raw materials Rs. 380, Labour charges Rs. 228, Others Rs. 152 (ii) Rs. 192·66 **31.** Buffalo Rs. 2200, Ox Rs. 800 **32.** 16% **33.** Rs. 380, Rs. 1140 **34.** (i) 8 (ii) Rs. 24·15 (iii) Rs. 12·25 (iv) 26·25 (v) Rs. 6 **35.** Rs. 1929·73 (approx.) gain **36.** Rs. 310, Rs. 490.

CHAPTER 1

EXERCISES ON PREVIOUS LESSON

Exercise 1

1. Multiply the following :

(i) $2x + 3y + 15, 3x + 5y + 1$

(ii) $5a^2 - 6a + 8, 4a^2 + 2a - 5$

(iii) $a^4 + a^2 + 1, a^4 - a^2 + 1$

(iv) $8m^3 - 3m - 2m^2, 3m^2 - 5m + 1$

(v) $\frac{1}{3}x^2 + \frac{1}{2}xy + y^2, \frac{1}{4}x - \frac{1}{3}y$

2. Find the continued product of the following :

(i) $a - b, a^2 + ab + b^2, a^3 + b^3$

(ii) $x^4 + x^2 + 1, x^4 - x^2 + 1, x^8 - x^4 + 1$

(iii) $a - b, b - c, c - a$

(iv) $p - q, p + q, p^2 + q^2, p^4 + q^4, p^8 + q^8$

(v) $a + b + c, a + b - c, a - b + c, b + c - a$

3. Multiply the following by the method of detached coefficient :

(i) $x^4 + x^3 + 2x - 1, x - 1$

(ii) $2x^2 + 9xy - 3y^2,$

4. Divide the first expression by the second in the following :

(i) $a^2 - 4ab + 4b^2 - 9c^2, a - 2b + 3c$

(ii) $x^3 - 2ax^2 + 4a^2x - 8a^3, x - 2a$

(iii) $x^2 + 4y^2 + 9z^2 + 4xy + 12yz + 6zx,\ x + 2y + 3z$

(iv) $6a^4 - 11a^3b - 8a^2b^2 + 22ab^3 - 8b^4, a^2 - 2b^2$

(v) $p^3 + q^3 + 8r^3 - 6pqr, p + q + 2r$

(vi) $p^5 + p - 1, p - 1$

(vii) $3x^4 - 5x^3 - 2x^2 + 5x + 3, 1 + x - x^2$

(viii) $4x^4 + 12x^2 + x + 18, 2x^2 - 2x + 4$

(ix) $m^4 + 2m^3 + 3m^2 - 2m + 9, m^2 + m + 1$

(x) $y^4 + \frac{5}{12}y^2 + \frac{1}{2}y - 1, y^2 + \frac{1}{2}y + \frac{1}{3}$

5. Divide the first expression by the second using the method of detached coefficient :

(i) $8l^3 + 36l^2m + 150lm^2 + 27m^3 , 4l^2 + 12lm + 9m^2$

(ii) $x^7 - 1, x - 1$

6. Multiply $x^2 - 3x - 3$ by $3x - 5$. Divide the product by $x - 3$.

7. Divide $4x^2 - 11x + 7$ by $4x - 7$. Multiply the quotient by $3x - 8$.

8. $\frac{7a^2 - 20ab + 12b^2}{a - 2b} \times (3a - 2b) = ?$

(Hint : first divide $7a^2 - 20ab + 12b^2$ by $a - 2b$)

9. $\frac{p^2 + 10pq + 16q^2}{p + 8q} \times \frac{p^2 - 10pq + 16q^2}{p - 8q} = ?$

Exercise 2

1. Find the cube of the following using formula :

(i) $1 - 3a$ (ii) $2x + 3y$ (iii) $a + b + c$

(iv) $1 - x - y$ (v) 999 (vi) 1001

2. Express the following in Simplest form :

(i) $(5x - 3y + 7z)^3 - 3(5x - 3y + 7z)^2(7z - 3y) + 3(5x - 3y + 7z)(7z - 3y)^2 - (7z - 3y)^3$

(ii) $28{\cdot}63 \times 28{\cdot}63 \times 28{\cdot}63 + 3 \times 28{\cdot}63 \times 28{\cdot}63 \times 21{\cdot}37 + 3 \times 28{\cdot}63 \times 21{\cdot}37 \times 21{\cdot}37 + 21{\cdot}37 \times 21{\cdot}37 \times 21{\cdot}37$

(iii) $\frac{(2x+3y)^3+3(2x+3y)\ 2(3z-2y)+3(2x+3y)\ (3z-2y)^2+(3z-2y)^3}{(x-y)^2+2(x+y)(x+3z)+(x+3z)^2}$

(iv) $\frac{(a+b)^2+3(a+b)^2(a-b)+3(a+b)(a-b)^2+(a-b)^2}{(a+b)^3-3(a+b)^2(a-b)+3(a+b)(a-b)^2-(a-b)^2} \times \frac{(a+b)^2-2(a+b)(a-b)+(a-b)^2}{(a+b)^2+(a+b)^2(a-b)+(a-b)^2}$

3. (i) If $a + 2b = 5$ and $a^2 + 4b^2 = 13$, find ab.

(*ii*) If $9a^2 + 4b^2 = 2$ and $3a + 2b = 2$, find ab.

(*iii*) If $p - q = 6$ and $p^3 - q^3 = 936$, find pq.

(*iv*) If $p = 4$, find the value of $p^3 - 3p^2 + 3p - 1$.

(*v*) If $3l - 4m = 5$, find $27l^3 - 64m^3 - 180lm$.

(*vi*) If $y + z = 4$ and $yz = 3$, what is the value of $y^3 + z^3$?

(*vii*) If $a^3 + b^3 = m$ and $a + b = m$, express ab in terms of m and n.

(*viii*) If $2p + 3q = 5$ and $pq = 1$, find $8p^3 + 27q^3$.

(*ix*) If $l + m + n = 9$ and $l^2 + m^2 + n^2 = 31$, what is $lm + mn + nl$?

(*x*) If $s + t = 12$ and $s - t = 8$, find the values of the following expressions :

$$st,\ s^2 - t^2,\ s^2 + t^2,\ s^3 + t^3,\ s^3 - t^3$$

4. (*i*) If $x + \frac{1}{x} = 5$ find the values of the following :

$$x^2 + \frac{1}{x^2},\ \left(x - \frac{1}{x}\right)^2,\ x^3 + \frac{1}{x^3},\ x^4 + \frac{1}{x^4}$$

(*ii*) If $p^2 + \frac{1}{p^2} = 47$, find the values of $\left(p + \frac{1}{p}\right)$, $\left(p - \frac{1}{p}\right)^2$ and $p^4 + \frac{1}{p^4}$

(*iii*) Given $3p + \frac{1}{5p} = 4$, find the values of $9p^2 + \frac{1}{25p^2}$

$$27p^3 + \frac{1}{125p^3} \text{ and } 81p^4 + \frac{1}{625p^4}$$

(*iv*) Given $2x - \frac{1}{3x} = 5$, find the values of $4x^2 + \frac{1}{9x^2}$

and $8x^3 - \frac{1}{27x^3}$

5. By the help of formulae multiply the following :

(*i*) $(5x + 4y)(25x^2 + 40xy + 16y^2)$

(*ii*) $(xy - 3z)(x^2y^2 - 6xyz + 9z^2)$

(*iii*) $(x^3 - 1)(x^6 + x^3 + 1)$

(*iv*) $(p^2q^2r^2 + s^2)(p^4q^4r^4 - p^2q^2r^2s^2 + s^4)$

(*v*) $(m + n)(m^2 - mn + n^2)(m - n)(m^2 + mn + n^2)$

(*vi*) $(a + 5b)(a - 5b)(a^2 + 25b^2)(a^4 + 625b^4)$

(vii) $(p + \frac{1}{q})^3 \times (p - \frac{1}{q})^3$

(viii) Given $D = 14{\cdot}3$, $d = 13{\cdot}7$ and $\pi = \frac{22}{7}$, find the value of $\frac{1}{4}\pi D^2 - \frac{1}{4}\pi d^2$

Exercise 3

1. Factorise the following using formula :

(*i*) $a^{16} - b^{16}$, (*ii*) $1 - x^{12}$ (*iii*) $\frac{x^4}{16} - \frac{y^4}{81}$

(*iv*) $(a + b)^2 - 16(a - 2b)^2$ (*v*) $t^9 - \frac{1}{512}$

(*vi*) $AR^3 - Ar^3 + AR^2h - Ar^2h$

(*vii*) $m^2 + \frac{1}{m^2} + 2 - 2m - \frac{2}{m}$

(*viii*) $4x^2 - 12xy + 9y^2 + 2x - 3y$

(*ix*) $9p^2 - 24pq + 16q^2 + 3ap - 4aq$

(*x*) $\frac{x^3}{64} - \frac{64}{x^3}$ (*xi*) $n^2 - (m^2 - 2n^2)^2$ (*xii*) $729p^6 - q^6$

(*xiii*) $7x^7 - 112x^3y^4$ (*xiv*) $8(a+b)^3 - 27(b+c)^3$

(*xv*) $x^8 + \frac{m^4}{16} + \frac{1}{256}$

2. Resolve into factors by breaking the coefficient of the middle terms :

(*i*) $a^2 - 15ab + 54b^2$ (*ii*) $x^2 + 6x - 27$

(*iii*) $x^2 + xy - 110y^2$ (*iv*) $x^2 + 5xy - 176y^2$

(*v*) $x^2 - 9x - 486$ (*vi*) $5x^2 - 27xy + 10y^2$

(*vii*) $14x^2 + x - 3$ (*viii*) $9a^2 - 93a + 108$

(*ix*) $5 - 13p + 6p^2$ (*x*) $-12m^2 - 5m + 3$

(*xi*) $9 + 9x - 4x^2$ (*xii*) $(2a + b)^2 - 11(2a + b) + 28$

(xiii) $(p - q)^2 + 8(p - q)(p + q) + 12(p + q)^2$

(*xiv*) $2(x + y)^2 - 5(x + y) - 25$ (*xv*) $p^2 + p - (a + 1)(a + 2)$

3. **Resolve into factors :**

(*i*) $a^3 - 7a - 6$ (*ii*) $x^3 - 12x - 16$

(*iii*) $x^2 + 3x - a^2 - a + 2$

(iv) $(x - 1)(x - 2)(x + 3)(x + 4) + 6$

(v) $x^2 + (p + \frac{1}{p})\ x + 1$

Exercise 4

1. Find the H.C.F. of the following using factors :

(*i*) $x^3 - x^2, a(x - 1)^2, ax^5 - ax^2$

(*ii*) $a^4 - x^4, a^2 + 5ax + 4x^2$

(*iii*) $2a^2 + 11ab + 15b^2, a^3 + 27b^3$

(*iv*) $2a^2b + ab^2 - 2abc, 4a^2 + b^2 - 4c^2 + 4ab$

(*v*) $3p^2 - 8p + 4, 2p^2 - 5p + 2$

(*vi*) $4x^5 + 16x^4 - 20x^3, x^4 + 125x, 3x^4 + 14x^3 - 5x^2$

(*vii*) $x^2 - 3x + 2, x^3 - 1, x^2 - 9x + 8$

(*viii*) $5x^2 + 12x + 4, x^4 - 16, 3x^2 + 8x + 4$

(*ix*) $p^2 + 2pq + q^2 - r^2, p^2 - q^2 + pr - qr$

(*x*) $4p^2 + 12p + 9, 8p^3 + 36p^2 + 54p + 27, 8p^3 + 27$

(*xi*) $m^2 - 7m + 10, m^2 - 37m + 70, m^3 - 4m^2 + 4m$

(*xii*) $3p^3 - 3q^3, 2p^2 - 4pq + 2q^2, p^3 - 3p^2q + 3pq^2 - q^3$

2. Find the H.C.F. of the following by division method :

(*i*) $x^3 + x^2 + x + 1, x^2 + 2x + 1$

(*ii*) $x^3 + 5x^2 + 9x + 9, 2x^3 + 3x^2 + 4x - 3$

(*iii*) $2p^3 - 5p^2 + 5p - 3,\ 3p^3 - 5p^2 + 5p - 2$

(*iv*) $m^4 - 4m^3 + 3m^2 + 2m - 12, m^4 + 2m^2 + 9$

(*v*) $x^3 - 5x^2 - 2x + 24, x^3 - 9x^2 + 26x - 24$

(*vi*) $x^4 + x^3y - xy^3 + x^2y^2 - 2y^4, x^4 + 3x^2y^2 + 4y^4$

(*vii*) $3m^3 - 5m^2 + m + 1, 2m^3 - 7m^2 + 4m + 1$

3. Given $A = 8(a^2 - b^2), B = 12(a^4 - b^4), C = 18(a^4 - ab^3)$

Find the H.C.F's of A, B and B, C separately. Hence from these two H.C.F's, find the H.C.F. , A,B and C.

Exercise 5

1. **Find the L.C.M, of the following**
   - (i) $4x^3 + 4, 2x^2 + 4x + 2, 6x^3 + 18x^2 + 18x + 6$
   - (ii) $x^3 - 4x, x^4 - 3x^3 + 2x^2, x^5 - x^4 - 6x^3$
   - (iii) $x^2 - y^2 + z^2 + 2zx, x^2 - y^2 - z^2$ $+ 2yz, xy + zy + y^2 - z^2$
   - (iv) $4x^2 - 1, 8x^3 - 1, 16x^4 - 1, 64x^6 - 1$
   - (v) $x^2 + 9x + 20, x^3 + 64, x^2 + 7x + 12$
   - (vi) $x^2 - 12x + 20, x^2 - 23x + 130, x^2 - 9x + 14$
   - (vii) $y^3 - 19y + 30, y^3 + 3y^2 - 9y + 5$
   - (viii) $1 - m^2, 2 - 2m^3, 1 + m^2 + m^4,$ $m^2 + 3m + 2, m^3 + m^2 + m$
   - (ix) $p^3 - 6p^2 + 11p - 6, p^2 - 4p + 3$

2. Find the H.C.F. and the L.C.M. of $15x^2 + 26x - 21$ and $3x^2 - 23x - 70$. What is the product of these L.C.M. and H.C.F.? Verify whether this product is equal to the product of the two expressions.

3. Find the H.C.F. and the L.C.M. of $x^2 - 20x + 96$ and $x^2 - x - 56$. Find the product of these L.C.M. and H.C.F. and verify whether the second expression is obtained on dividing this product by the first expression.

4. The H.C.F. and the L.C.M. of two expresions are respectively $x - 3$ and $x^3 - 13x^2 + 14x + 48$. What is the product of the expressions ? What are the expressions ?

5. The product of two quantities is $x^4 - 3x^3 + 17x^2 - 17x + 2$. If their H.C.F. be $x - 1$, what is their L.C.M. ?

6. The L.C.M. and H.C.F. of two quantities are respectively $6x^3 + 35x^2 - 7x - 6$ and $2x - 1$. If one of the quantities be $6x^2 - x - 1$, find the other.

Exercise 6

Express the following into the simplest form .

1. $\dfrac{1}{a-b}-\dfrac{1}{a+b}-\dfrac{4ab}{2a(a^2-b^2)}$

2. $\dfrac{a+1}{a-2}\times\dfrac{a^2-a-2}{a^2+a}$

3. $\dfrac{x-y}{y}-\dfrac{x}{x+y}-\dfrac{x^3-x^2y}{x^2y-y^3}$

4. $\dfrac{b^2-5b}{3b-4b}\times\dfrac{9b^2-16a^2}{b^2-25}\times\dfrac{ab+5a}{3b^2+4ab}$

5. $\dfrac{a}{(b-a)(a-c)}+\dfrac{b}{(c-b)(b-a)}+\dfrac{c}{(a-c)(c-b)}$

6. $\dfrac{p^2-q^2}{m+n}\div\dfrac{p-q}{m+n}$ 7. $\dfrac{l^2-m^2}{l^3-m^3}\times\dfrac{l^2+lm+m^2}{l^2+2lm+m^2}$

8. $\dfrac{x^2-x-2}{x^2-x-6}\div\dfrac{x^2-2x}{2x+x^2}$

9. $\dfrac{x^2+y^2-z^2-2xy}{x^2+y^2-z^2+2xy}\div\dfrac{x-y+z}{x+y-z}$

10. $\dfrac{b-c}{a^2-(b-c)^2}+\dfrac{c-a}{b^2-(c-a)^2}+\dfrac{a-b}{c^2-(a-b)^2}$

11. $\dfrac{(x-z)^2-y^2}{x^2-(y+z)^2}+\dfrac{(y-x)^2-z^2}{y^2-(z+x)^2}+\dfrac{(z-y)^2-x^2}{z^2-(x+y)^2}$

Exercise 7

1. Solve the following equations :

$$\frac{2x-2}{5}-4x=\frac{1-x}{2}-x-24$$

$$\frac{5}{8}(7x+5)-7\frac{3}{8}=13+\frac{3}{4}(5x+2)$$

$$\frac{10x}{9}-\frac{4x-1}{7}-\frac{3x-2}{5}=0$$

$$\frac{1}{x-1}+\frac{1}{x-4}=\frac{2}{x-3}$$

$$\frac{p}{p-x}-\frac{q}{q-x}=\frac{p+q}{p+q-x}$$

$$\frac{x-a}{b}+\frac{x-b}{a}+\frac{x-3a-3b}{a+b}=0$$

$$\frac{1}{x+3}+\frac{3}{2x+6}=\frac{1}{x+1}$$

$$\frac{3x+5}{x-2}+\frac{2x+3}{x-1}=\frac{10x+5}{2x-4}$$

(In each case verify whether the value of x satisfies the equation or not)

Exercise 8

1. In the equation $\frac{C}{5}=\frac{F-32}{9}$ taking F as the known quantity find the value of C. If C is taken as the known quantity what will be the value of F when F is unknown ?
2. From the relation u + ft = v, express f in terms of u, v and t
3. From the relation $\frac{PV}{T}=R$ express T in terms of P, V and R.
4. From the relation $\frac{1}{R}=\frac{1}{r_1}+\frac{1}{r_2}$ express r_1 in terms of R and r_2.
5. From $I=\frac{nE}{R+nr}$ Express r in terms of n, E, I and R.
6. From $P=\frac{gs}{g+s}$ express s in terms of P and g.
7. From $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$, express f in terms of u and v.

CHAPTER 2

Formation of Simultaneous Equations

In class VII you have learht to form simple linear equations. Here we shall take a few problems of this type and discuss elaborately.

Let us take problem 8, exercise 23 on page 107 of class VII Algebra portion. The problem reads as follows :

"5 times of which number is added to 15 to give 6 times that number ?"

It is easily understood from the arithmetic point of view that the number is 15. But here we shall not solve this problem, only we shall try to form a simple equation which on being solved will give the number.

To form the equation observe that in this problem there is one unknown quantity which is a number. An other condition is imposed involving the value of this unknown quantity. The condition is "5 times this unknown when added to 15 gives 6 times this unknown." In general, we assume the unknown quantity as x. Then this x is expressed with the help of the condition of the problem in mathematical language and we get a relation. Here the relation is

$$5x + 15 = 6x$$

This is broadly the method of forming an equation. Sometimes a partial change in the method is required, e.g. let us consider the next problem of the same exercise.

"In an examination the marks in English of Monowara is 7 more than that of Indrani and the sum of their marks is 117. Find the scores of each."

Actually there are two unknown quantities, here, one is the marks in English of Indrani and the other is the marks in English of Monowara. Also there are two given conditions "viz. (i) Monowara has obtained 7 marks more than Indrani and (ii) the sum of their marks is

117. "In this case we can take two unknown quantities and form two equations. But that will not be a simple equation, since a simple equation involves only one unknown quantity.

Let the marks of in English Indrani be x then form the first condition, the marks of Monowara is $x + 7$ and the sum of their marks is $x + (x + 7) = 2x + 7$. Now from the second condition of the problem.

$$2x + 7 = 117$$

This is the required simple equation.

But if you take x and y to denote the two unknown quantities in this problem then using the two conditions you can form two equationas follow :

Let the marks in English which Indrani and Monowara obtianed be respectively x and y. Then from the first condition y is 7 more than x i.e. $y = x + 7$ and the sum of the two scores is 117 i.e., $x + y = 117$.

If we take these two equations simultaneously and write together, we get

$$y = x + 7$$
$$x + y = 117$$

These two equations are called simultaneous equations, because they together bear the solution of one problem. It is not possible to detach one of the simulataneous equations from the other, since none alone can give a definite solution of a certain problem.

In class VII you have learnt to form simple equations and to solve such simple equations. It is definitely known to you that the value of the unknown quantity is the solution of the equation.

Now the question arises whether in all cases a simple equation has one or more that one solution. The answer is quite clear that a simple equation has one and only one solution in all cases ; more than one solution is never possible. An example will clarify the fact.

The problem on Indrani and Monowara's Examination gave us the following equation $2x + 7 = 117$. Subtracting 7 from each side, we get $2x = 110$.

Now dividing both sides by 2, one gets $x = 55$.

This is the required solution. i.e. Indrani has secured 55 marks while Monowara secured $55 + 7 = 62$ marks and the sum of their marks is $55 + 62 = 117$.

If possible let x_1 and x_2 be two distinct solutions of problem. Then

$$2x_1 + 7 = 117 \quad \ldots \text{(i)}$$

and

$$2x_2 + 7 = 117 \quad \ldots \text{(ii)}$$

Subtracting relation (i) from (ii) we have $2(x_2 - x_1) = 0$. Dividing both sides by 2 we find $x_2 - x_1 = 0$ or $x_2 = x_1$ i.e., the solutions are not distinct but same.

Now what can you say about the solution of simultaneous equations ? Later on you will learn the method of solving, simultaneous equations of Indrani-Monowara problem are :

$$y = x + 7$$

and

$$x + y = 117$$

It is already mentioned that in these two simultanceous equations one cannot be detached from the other, as in that case, it will neither be a simple equation nor be simultaneous equations, and that will also not bear a definite solution of the problem. If we consider them simultaneously and if they are different and consistent, then we shall find that x and y have one and only one pair of values which sa tisfy the two equations simultaneously.

This pair of values of x and b y are that called the unique solution of the equation. You will learn later how these values can be obtained. Now we can show that there is not two distinct pair of values of x and y. If possible let x_1, x_2 and x_2, y_2 be two distinct pair of values which satisfy the equations, then we have

$$y_1 = x_1 + 7 \qquad \ldots (1)$$

$$x_1 + y_1 = 117 \quad \text{... (2)}$$

$$y_2 = x_2 + 7 \quad \text{... (3)}$$

$$x_2 + y_2 = 117 \quad \text{... (4)}$$

Subtracting Equ (3) from Equ (1) we get

$$y_1 - y_2 = x_1 - x_2 \quad \text{... (5)}$$

Subtracting Equ (4) from Equ (2) we get

$$(x_1 - x_2) + (y_1 - y_2) = 0 \quad \text{... (6)}$$

Substituting (5) in (4) from equ (5) and equ (6)

$$2(x_1 - x_2) = 0$$

$$\text{i.e., } x_1 = x_2$$

Again, using equ (7) in equ (5), we have $y_1 = y_2$

The pair x_1, y_1 coincide with x_2, y_2 and they are not distinct.

Earlier it is mentioned that the equations should be different and consistent. If they are not different we cannot find a unique solution,

e.g., the equations $x + y = 117$

$$2x + 2y = 234$$

are not different but same, since if the former is multiplied by 2, the latter is obtained. In this case there is no distinct solution.

Again, observe that the equations

and $x + y = 117$

$$x + y = 119$$

are inconsistent, since if one is ture the other is not. Here also no definite solution is possible.

Example 1. Express the conditions given in the following problem in terms of equations :

"The cost of 3 pencils and 2 pens together is Rs. 7 and that of 4 pencils and 3 pens together is Rs. 10. Find the cost of a pencil and a pen.

Solution. Here we observe that in the problem the unknown quantities are the cost of a pencil and the cost of a pen. So we can assume,

the cost of a pencil is Rs. x.

and the cost of a pen is Rs. y.

The two given conditions in the problem are as follows :

First condition is :

the cost of 3 pencils and 2 pens together is Rs. 7.

Second condition is :

The cost of 4 pencils and 3 pens together is Rs. 10.

Now the cost of 3 pencils is Rs. $3x$ and the cost of 2 pens is Rs. $2y$. So the price of three pencils and two pens together is Rs. $(3x + 2y)$.

Using '=' sign we can write it as

$$3x + 2y = 7 \qquad \ldots \quad (1)$$

This is an equation.

In a similar way from the second condition, we get

$$4x + 3y = 10 \qquad \ldots \quad (2)$$

Hence from the given problem, we get the two equations (1) and (2).

Example 2. Express the following statements in the form of equations :

(i) In a certain fraction if 1 is taken away from the numerator and 2 is added to the denominator, it becomes equal to $\frac{1}{2}$ and when 7 and 2 are taken away from the numerator and denominator respectively, the fraction becomes $\frac{1}{3}$.

(ii) 20 years ago, the age of the father was 4 times that of the son, while 4 years hence, father's age will be twice the age of the son.

(iii) The perimeter of a rectangular courtyard is 60 metre. If its length is increased by 3 metres and the width is decreased by 3 metres its area decreases by 21 sq. metres.

Solution. *(i)* Let x and y be the numerator and the denominator respectively of the fraction. Therefore, the fraction is $\frac{x}{y}$ From the first condition, if 1 is subtracted from the numerator and 2 is added to the denominator, the new fraction is $\frac{x-1}{y+2}$ and it is equal to $\frac{1}{2}$.

∴ we can write

$$\frac{x-1}{y+2} = \frac{1}{2}$$

$$2(x-1) = y+2$$

$$\text{or,}\quad 2x-2 = y+2$$

$$\text{or,}\quad 2x-y = 4 \quad \ldots \quad (1)$$

Again, from the second condition of the problem, if 7 and 2 are taken away from the numerator and the denominator respectively the fraction will be The value of this fraction is given as so we can write

$$\frac{x-7}{y-2} = \frac{1}{3}$$

$$\therefore\ 3(x-7) = y-2$$

$$\text{or,}\quad 3x-21 = y-2$$

$$\text{or,}\quad 3x-y = 19 \quad \ldots \quad (2)$$

Hence the required equations are

$$2x-y=4$$

$$3x-y=19$$

(ii) Let the present age of father be x years and that of the son be y years.

20 years ago the age of the father was $(x-20)$ yrs. and the age of the son was $(y-20)$ yrs.

From the 1st condition the age of the father 20 years ago was 4 times the age of the son, i.e. if we multiply son's age 20 years ago by 4, that will be the father's age at that time.

$$\therefore \quad x - 20 = 4\,(y - 20) \qquad \ldots \qquad (1)$$

Again 4 years hence the age of the father is $(x + 4)$ yrs. and the age of the son is $(y + 4)$ yrs. Now from the second condition, the age of father is twice the age of the son four years hence.

$$\therefore \quad x + 4 = 2(y + 4) \qquad \ldots \qquad (2)$$

Hence the required equations are

$$x - 20 = 4(y - 20)$$

and $$x + 4 = 2(y + 4)$$

(*iii*) Let the length $= x$ metres

and the breadth $= y$ metres

$\therefore$ perimeter $= 2\,(x + y)$ metres.

But in the 1st condition the perimeter is given to be equal to 60 metres.

$$\therefore \quad 2(x + y) = 60 \qquad \ldots \qquad (1)$$

Again when the length is x metres and the breadth is y metres the area becomes xy sq. metres.

From the second condition of the problem when the length is increased by 3 metres and the breadth is decreased by 3 metres, the length becomes $(x + 3)$ metres and the breadth $(y - 3)$ metres whence the area is $(x + 3)\,(y - 3)$ sq. metres. It is given that this new area is 21 sq. metres less than the former.

$$\therefore \quad (x + 3\,)\,(y - 3) = xy - 21 \qquad \ldots \qquad (2)$$

Hence the required equation are

$$2(x + y) = 60$$

and $$(x + 3)\,(y - 3) = xy - 21$$

Example 3. From the following information write down the equations for finding out the speed of the boat and the current:

"A boat moves 48 km in 16 hrs along the current while it moves 16 km in 8 hrs against the current."

Solution. Let the speed of the boat in still water is $(x + y)$ km per hour and the speed of the boat against the current is $(x - y)$ km per hour. Hence, along the current the boat moves $16\,(x + y)$ km in 16 hrs and against the current it moves $8\,(x - y)$ km in 8 hrs.

$$\therefore \quad 16\,(x + y) = 48 \qquad \ldots \qquad (1)$$

$$\text{and} \quad 8(x - y) = 16 \qquad \ldots \qquad (2)$$

Hence the required equations are

$$16(x + y) = 48$$

and $$x - y = 2$$

Example 4. Express the following statement in the form of the equations :

"In a number of two digits the sum of the digits is 7. If the digits are interchanged the new number thus formed is 27 less than the former".

Solution. Let the digit in the unit's place be x and that in the ten's place be y. Then the number is

$$10y + x$$

From the first condition, we find

$$x + y = 7 \qquad \ldots \qquad (1)$$

If the digits are interchanged the digit in the unit's place is y and that in the ten's place becomes x and the new number formed is $10x + y$

From the second condition of the problem the new number is 27 less than the original.

$$\therefore \quad 10x + y = 10y + x - 27 \qquad \ldots \qquad (2)$$

$\therefore$ the required equations are

$$x + y = 7$$

and $$(10y + x) - (10x + y) = 27$$

Exercise 9

Express the following statements in the form of equations :

1. The sum of two numbers is 60 while their difference is 10.

2. Among two numbers if 3 times the second is taken away from twice the first, the result is 15 and when twice the second is taken away from the first the result is 6.

3. In a fraction if 1 is added to the denominator, it becomes $\frac{1}{2}$ and when 1 is taken away from the numerator it becomes $\frac{1}{3}$.

4. The sum of two numbers is 80 and three times their difference is 20 more than the greater number.

5. The sum of the digits of a number between 10 and 100 is 8. If 45 is taken away from that number, it becomes equal to a number which is formed by interchanging the digits of the original number.

6. The cost of 3 kg of tea and 4 kg of sugar together is Rs.80. If the price of tea is decreased by 10% and that of sugar is increased by 10% then the cost of above tea and sugar together becomes Rs. 76.

7. The perimeter of a rectangular garden is 60 metres; if its length is increased by 3 metres and breadth is decreased by 3 metres, its area is decreased by 21 sq. metres.

8. The sum of the digits of a two-digit number is 54 less than the number. If the digits are interchanged the number thus formed is 27 more than the original number.

9. From two places 60 km apart *A* and *B* started at the same time moving towards each other and met each other after 6 hrs. If *A* moved with $\frac{2}{3}$ of his speed and *B* with twice his speed, they would have met 5 hours after they started.

10. While distributing some mangoes to certain number of boys it is found that if each was given 2 mangoes, there would be an excess of 4 mangoes, but if they were provided with 3 mangoes each there would be a shortage of 5 mangoes.

11. A person can row 30 km in 5 hrs along the current and 16 km in 4 hrs against the current.

12. 4 years ago the ratio of ages of Saswati and Srabanti was 3 : 2. 4 years hence the ratio of their ages will be 7 : 5.

13. A boat can move 30 km upstream and then 44 km downstream in 10 hours and can move 40 km upstream and then 55 km downstream in 13 hours.

14. With a sum of money Rahim buys some mangoes. If he got 10 mangoes more, then each would cost 2 paise less and if he got 15 mangoes less then each would cost 6 paise more.

15. Dipankar reached his destination moving at the usual speed. He would have taken 2 hours less moving at a speed of 2 km more and would have take 8 hrs more moving at a speed of 2 km less per hour.

CHAPTER 3

Solution of Linear Simultaneous Equations

You have seen in previous chapter how simultaneous equations are formed from a practical problem. In a problem if there are two unknown quantities and two independent conditions. then taken x and y as the unknowns we can form two simultaneous equations. A definite solution is not possible from one linear equation with two unknowns.

Two linear simultaneous equations with two unknown quantities can be solved by different methods. We shall now discuss them one by one.

For solving a simple equation, it is known to us that any quantity may be added to or subtracted from either side of the equation or either side of that equation may be multiplied or divided by any non-zero quantity. These rules are valid for every equation of the simple simultaneous equations.

The general form of a linear equation with two unknown quantities is $ax + by + c = 0$. Hence a,b,c may have any arbitrary values. Following are some examples of linear equations with two unknowns

$-3x + 4y + 5 = 0$; $5x - 3y + 6 = 0$; $2x + 3y + 7 = 0$ etc.

Again, $3x = 4y + 5$, $x = \frac{4y + 5}{3}$, $4y = 3x - 5$, $y = \frac{3x - 5}{4}$ are different representations of the same linear equation $-3x + 4y + 5 = 0$. For solving simple simultaneous equations their different forms will be considered as is done in the case of simple equations.

Different methods of solving linear simultaneous equations are (*i*) method of comparison (*ii*) method of substitution, (*iii*) method of elimination and (iv) method of cross multiplication.

I. Method of Comparison :

Let us consider the following two equations for their solution.

$$7x - 3y = 31 \qquad \ldots \qquad (1)$$

$$9x - 5x = 41 \qquad \ldots \qquad (2)$$

From each of the equations (1) and (2), x can be expressed in terms of y.

Form equation (1), $7x = 3y + 31$

$$\text{Or, } x = \frac{3y + 31}{7} \qquad \ldots \qquad (3)$$

Again from equation (2), $9x = 5y + 41$

$$\text{Or, } x = \frac{5y + 41}{9} \qquad \ldots \qquad (4)$$

Here equations (3) and (4) are the new forms of equations (1) and (2). There are infinite values of x and y which will satisfy equation (3) alone. When $y = 1$, the value of x is $4\frac{6}{7}$. Similarly, when y has the values 2, 3, 4, 5, ...–1, –2, –3, –4, –5, ... the corresponding values of x are expressed in the following table:

x	$4\frac{6}{7}$	$5\frac{2}{7}$	$5\frac{5}{7}$	$6\frac{1}{7}$	$6\frac{4}{7}$	4	$3\frac{4}{7}$	$3\frac{1}{7}$	$2\frac{5}{7}$	$2\frac{2}{7}$
y	1	2	3	4	5	–1	–2	–3	–4	–5

Here only from equation (3) [i.e. equation (1)] the values of x, y are $(4\frac{6}{7}, 1)$, $(5\frac{2}{7}, 2)$, etc.

In a similar way from equation (4) taking the value of y as 1, 2, 3,.........., –1, –2,–3, ... etc, the corresponding values of x thus obtained are given in the following table :

x	$5\frac{1}{9}$	$5\frac{6}{9}$	$6\frac{2}{9}$	$6\frac{7}{9}$	$7\frac{7}{9}$	4	$3\frac{3}{9}$	$2\frac{4}{9}$	$2\frac{3}{9}$	$1\frac{1}{9}$
y	1	2	3	4	5	–1	–2	–3	–4	–5

Here only from equation (4) [i.e. equation (2)] the values of x, y are $(5\frac{1}{9}, 1), (5\frac{6}{9}, 2), (6\frac{2}{3}, 3)$,......... etc.

The values of x and y which simultaneously satisfy equations (3) and (4) will be the required solutions of the equations.

To get such values of x and y we have to find out the common solutions from the different sets of solutions of the two different equations. Is there any such pair of values of x and y which satisfy the above two equations at a time ? Minutely observe the two tables, you will find that if the corresponding values of *x* and *y* respectives 4 and -1, both the equations will see satisfied. Hence the corresponding solutions of (3) and (4) [i.e. (1) and (2)] will be $x = 4$ and $y = -1$.

Of course, it is true that we have not written all the different solutions of equations (3) and (4) separately. Taking the values of y as 6, 7, 8....., -6, -7, -8, ... find the corresponding values of x and examine whether x and y have any other common solutions or not. You will find no such common values exist.

It is difficult to find general values of *x* and *y* from such infinite solutions and if the general values of *x* and *y* be fractions then solving from such tables will be a very complicated matter. In this case we will have to known the general methods of solving such simultaneous equations.

Can we assume that the two values of *x* obtained from equations (1) and (2) are equal ? In other words can we assume that value of $x \left(= \frac{3y + 3}{7} \right)$ from equation (3) and the value of $x \left(= \frac{5y + 41}{9} \right)$ from equation (4) are equal ? To assume them equal means that for particular value of x, the definite value of y thus obtained form an equation involving *y* only,

i.e.,
$$\frac{3y + 31}{7} = \frac{5y + 41}{9} \qquad \dots \qquad (5)$$

From equation (5) the value of y thus obtained together with the corresponding value of x will satisfy the two equations simultaneously and will give their general solution.

By the rules of solving a simple equation of one unknown we can solve equation (5) in the following way.

$$\frac{3y+31}{7}=\frac{5y+41}{9}$$

Or, $27y + 279 = 35y + 287$ (By cross multiplication)

or, $27y - 35y = 287 - 279$

or, $-8y = 8$

$\therefore\ y = -1$

Now, to get the corresponding value of x, we substitute the value of y in (3) or (4),

i.e. $x = \frac{3x\ (-1) + 31}{7} = 4$

$\therefore$ We can say that, x = 4 and y = –1 are the general solutions of both the equations.

We can get the value of x by putting the value of y in equation (1) or (2) also.

In a similar way from equations (1) and (2) express y in terms of x. In the two equations thus obtained, substitute for x the values 1, 2, 3, 4, 5,....., –1, –2, –3, –4, –5...... etc, find the corresponding values of y and make two tables as before. From these two tables do you get any common solution of x and y ? Are the previous solution ($x=4, y=-1$) and the present solution you obtain same ?

Thus the general rule for solving two simultaneous equations are as follows :

(1) (i) from each of the two equations you have to express *x* in terms of *y*,

(ii) the two values of *x* thus obtained should be taken to be equal,

(*iii*) from the simple equation thus obtained in terms of *y*, the value of *y* has to be found out,

(iv) substitute this value of y in any one of the two simultaneous equations to find the value of x,

(v) the value of x and y thus obtained will be the solution of the simultaneous equations.

(2) In a similar way from the two equations y can be expressed in terms of x and these two values of y is equated so that a simple equation in x in formed. Solving this equation for x and using this value of x in any one of the two simultaneous equations the value of y is found out. In this way the value of y is found out. In this way by finding the values of x and y, the two equations can be solved.

The method of solving two simultaneous equations by comparing the values of x and y is called *comparison method.*

Now observe the solution of the following equations by this method.

Example 1. Solve $3x - 2y = 2$ ... (i)

$7x + 3 = 43$ ... (ii)

Instructions	**Method of solution**
(a) From (i) express x in terms of y In a similar way, from (ii) express x in terms of y	From (i) we get $3x = 2y + 2$ $\therefore \quad x = \frac{2y+2}{3}$ ..(iii)
(b) Taking the values of x in (iii) and (iv) to be equal, write the equation in terms of y	From (ii) we get $7x = -3y + 43$ $\therefore \quad x = \frac{-3y + 43}{7}$ .. (iv)
(c) Solve this simple equation in y	From (iii) and (iv), we have $\frac{2y+2}{3} = \frac{-3y + 43}{7}$ .. (v)
(d) By substituting the value of y in (ii) or (iv) find the value of x	By cross-multiplication we get $14y + 14 = -9y + 129$ or, $14y + 9y = 129 - 14$ or, $23y = 115$ $\therefore \quad y = 5$

Instructions	Method of solution
(e) Now write down the required solution of the equations	$x = \frac{2y+2}{3} = \frac{10+2}{3} = \frac{12}{3} = 4$ The required solution is $\left.\begin{matrix} x = 4 \\ y = 5 \end{matrix}\right\}$

Example 2. Solve

$$x + 5y = 36 \quad \ldots \quad (1)$$

$$x - 4y = 0 \quad \ldots \quad (2)$$

from eqn (1) we get $x = 36 - 5y$ ... (3)

and from eqn (2), we get $x = 4y$ ... (4)

From equs (3) and (4) by comparing the values of x, we have

$4y = 36 - 5y$ or, $4y + 5y = 36$

$9y = 36$ or, $y = \frac{36}{9} = 4$

In (2) substituting 4 for y we get

$x - 4.4 = 0$ or, $x = 16$.

The required solution is $\left.\begin{matrix} x = 16 \\ y = 4 \end{matrix}\right\}$

Example 3. solve

$$\frac{1}{3}(x - y) = \frac{1}{4}(y - 1)$$

$$\frac{1}{7}(4x - 5y) = x - 7$$

Here at first the two equations should be expressed in simplified form and then by the above rule they have to be solved.

$\frac{1}{3}(x - y) = \frac{1}{4}(y - 1)$

or, $4(x - y) = 3(y - 1)$

or, $4x - 4y = 3y - 3$

or, $4x - 4y - 3y = -3$

or, $4x - 7y = -3$ ... (1)

Again, $\frac{1}{7}(4x - 5y) = x - 7$

or, $4x - 5y = 7(x - 7)$
or, $4x - 7x - 5y = -49$
or, $-3x - 5y = -49$
or, $3x + 5y = 49$ $\quad\dots \quad (2)$

From eqn (1), we get $4x = 7y - 3$ or, $x = \frac{7y-3}{4}$ $\quad\dots \quad (3)$

and from eqn (2), we get $3x = 49 - 5y$ or,

$$x = \frac{49-5y}{3} \qquad \dots \qquad (4)$$

Now comparing the values of x from eqns (3) and (4), we have

$$\frac{7y-3}{4} = \frac{49-5y}{3}$$

$$\text{or, } 21y - 9 = 196 - 20y$$

$$\text{or, } 41y = 205$$

$$\therefore y = \frac{205}{41} = 5$$

Substituting $y = 5$ in eqn (3), we get

$$x = \frac{7.5-3}{4} = \frac{32}{4} = 8$$

$\therefore$ the required solution is $\left.\begin{matrix} x = 8 \\ y = 5 \end{matrix}\right\}$

Exercise 10

1. In each of the cases given below there are two simultaneous equations. From each of the equations express x in terms of y and taking the value of y as 1, 2, 3, 4, –1, –2, –3, –4, find the corresponding values of x and complete the table as given below.

From the table is it possible to get a common solution of the equations ?

x								
y	1	2	3	4	– 1	– 2	– 3	– 4

(i) $3x - 2y = 0$
$4x + 3y = 17$

(ii) $3x - 2y = 8$
$4x + 3y = 5$

(iii) $x - y = 9$
$2x - 3y = 2$

(iv) $x + y = 0$
$2x - 3y + 25 = 0$

2. From each of the simultaneous equations given in the above exercises express y in terms of x. Taking the values of x as 1, 2, 3, 4, –1, –2, –3, -4, –5, find the corresponding values of y and complete the following table. Can you get a common of the simultaneous equations ?

x	1	2	3	4	–1	–2	–3	–4	–5
y									

3. Solve by the method of comparison the following equations :

(i) $4x - 5y = 7$, $2x + 3y = 9$

(ii) $x - 9y = 8$, $7x + 2y = -9$

(iii) $7x - 3y = 31$, $9x - 5y = 41$

(iv) $13x - 12y + 15 = 0$, $8x - 7y = 0$

(v) $2x + 3y = 5$, $5x - 4y = -22$

(vi) $2x - 3y = 8$, $\frac{x + y}{x - y} = \frac{7}{3}$

(vii) $x + 2y = 3 = 4x - y$

(viii) $4x - y - 5 = 7x - 4y - 2 = 0$

(ix) $\frac{x + 1}{y + 1} = \frac{4}{5}$, $\frac{x - 5}{y - 5} = \frac{1}{2}$

(x) $x - 3 = 5(y - 3)$, $x + 4 = 7(y - 4)$

(xi) $x + y = 11$, $y + 2 = \frac{1}{8}(10y + x)$

(xii) $\frac{1}{x} + \frac{1}{y} = \frac{5}{6}$, $\frac{1}{x} - \frac{1}{y} = \frac{1}{6}$

(xiii) $\frac{x + y}{xy} = 2$, $\frac{x - y}{xy} = 1$

(xiv) $\frac{x}{3} = \frac{y}{2}$, $5x - 2y = 18$

(xv) $\frac{x + y}{5} + \frac{x - y}{4} = 5$, $\frac{x + y}{4} + \frac{x - y}{5} = 5\frac{4}{5}$

II. Method of Substitution :

Just like the method of comparison, in this method also a simple equation with one unknown quantity is formed from the two simultaneous equations. Here from any of the two equations the value of one unknown quantity is expressed in terms of the other and this value is substituted in the other equation.

Observe the following example,

Example 1. Solve

$$2x + y = 8 \qquad (i)$$
$$3x - 2y = 5 \qquad (ii)$$

non equation (i) we get $y = 9 - 2x$ (iii)

In eqation (ii) substituting $8 - 2x$ for y, equation which will not contain y but only x from this equation containing x. The value of x obtained will satisfy equations (i) and (ii) for a definite value of y.

In equations (i) for the values 0, 1, 2, 3, 4, –1, –2, –3, –4 of y, the corresponding values of x are given in the following table.

Values of y	Values of x from (iii)	Values of x from (ii)
0	4	$\frac{5}{3}$
1	$\frac{7}{2}$	$\frac{7}{3}$
2	3	3
3	$\frac{5}{2}$	$\frac{11}{3}$
4	2	$\frac{13}{8}$
–1	$\frac{9}{2}$	–1
–2	5	$\frac{1}{3}$
–3	$\frac{11}{2}$	$\frac{-1}{3}$
-4	6	–1

It is found that when $y = 2$, Then the values of x in both the equations are same and is equal to 3. From this we can say that when $x = 3$ and $y = 2$, both the equations are simultaneously satisfied.

It is difficult to find different values of x corresponding to different values of y from both the equations. So, we have to find out simple methods for solving these equations.

In equations (ii) and (iii) we can assume the values of y to be equal, since this value of y satisfies both the equations simultaneously.

In equation (ii) substituting $y = 8 - 2x$, we have

$$3x - 2(8 - 2x) = 5$$

$$\text{or} \quad 3x - 16 + 4x = 5$$

$$\text{or} \quad 7x = 5 + 16$$

$$\text{or} \quad 7x = 21$$

$$\text{or} \quad x = \frac{21}{7}$$

$$\therefore \quad x = 3$$

For this value of x, the corresponding values of y in both the equations will be equal. In equation (iii) substituting $x = 3$, we have

$$y = 8 - 2 \times 3 = 2$$

$\therefore$ the required solution is $x = 3, y = 2$

In a similar way from any equation we can express x in terms of y and substituting this value of x in the other equation the value of y can be found. You try yourselves in this way and see what the values of x and y are. Is there any difference between the present solution and the previous one ?

So the general rule in this case is as follows :

(1) First of all from any equation you have to express y in terms of x.

(2) This value of y has to be substituted in the other equation.

(3) Here the simple equation in x thus obtained when solved will give a value of x.

(4) This value of x when substituted in any one of the equations, the corresponding value of y will be obtained.

(5) These two values of x and y will be the solution of the given simultaneous equations.

(6) In a similar way from any equation expressing x in terms of y and substituting this value of x in the second equation y can be found out. Substituting the value of y in any one of the equations, x can be found and the solution of the simultaneous equations will be obtained.

Since in this form of solution, one unknown quantity is leaving substituted by an expression containg other unknown quantity, so this process is called the *method of substitution.*

Now observe the example given below :

Example 2.

$$7x - 3y = 31 \quad \text{.. (i)}$$

$$9x - 5y = 41 \quad \text{... (ii)}$$

Instruction	**Method of solution**
(a) From (i) express y in terms of x	From eqn. (i) we have $-3y = 31 - 7x$ or $3y = 7x - 31$ or $y = \frac{7x - 31}{3}$ ... (iii)
(b) In equation (ii) in place of y substitute the value of y obtained from eqn. (iii)	From (iii) substituting the value of y in eqn. (ii) we have $9x - 5\frac{(7x - 31)}{3} = 41$ ... (iv) Simplifying we get
(c) Now solve equation (iv)	$\frac{27x - 35x + 155}{3} = 41$ or $27x - 35x + 155 = 123$ or $-8x = 123 - 155$ or $-8x = -32$ or $x = 4$

Instruction	Method of solution
(d) Substitute the value of x in eqn. (iii) and determine the value of y	In eqn. (iii) putting the value of x we have $y = \frac{7 \times 4 - 31}{3}$ $= \frac{28 - 31}{3} = -1$
(e) Now write down the required solution of the two equations.	The required solution is $\left.\begin{array}{l} x = 4 \\ y = -1 \end{array}\right\}$

Now according to the above instructions observe the following solutions :

Example 3. Solve :

$$\frac{2}{x} + \frac{3}{y} = 2$$

$$\frac{5}{x} + \frac{10}{y} = 5\frac{5}{6}$$

From equation (i), we have

$$\frac{3}{y} = 2 - \frac{2}{x} \quad \text{or, } \frac{3}{y} = \frac{2x - 2}{x} \quad \text{or, } \frac{y}{3} = \frac{x}{2x - 2}$$

$$\therefore y = \frac{3x}{2x - 2} \qquad \ldots \qquad \text{(iii)}$$

In equation (ii) substituting $\frac{3x}{2x - 2}$ for *y*, we have

$$\frac{5}{x} + \frac{10}{\frac{3x}{2x - 2}} = \frac{35}{6}$$

IW or, $$\frac{5}{x} + \frac{10(2x - 2)}{3x} = \frac{35}{6}$$

$$\text{or, } \frac{1}{x} + \frac{2(2x - 2)}{3x} = \frac{7}{6}$$

$$\text{or, } \frac{3 + 4x - 4}{3x} = \frac{7}{6}$$

$$\text{or, } \frac{4x - 1}{3x} = \frac{7}{6}$$

(7) $\left.\begin{array}{l} y - x = 2 \\ 3x - 2y = 5 \end{array}\right\}$ (8) $\left.\begin{array}{l} 4x - 3y = 16 \\ 6x + 5y = 62 \end{array}\right\}$

(9) $\left.\begin{array}{l} \dfrac{2}{x} + \dfrac{5}{y} = 1 \\ \dfrac{3}{x} + \dfrac{2}{y} = \dfrac{19}{20} \end{array}\right\}$ (10) $\left.\begin{array}{l} 4x - 3y - 18 = 0 \\ 4y - 5x + 7 = 0 \end{array}\right\}$

(11) $\left.\begin{array}{l} 2x + \dfrac{3}{y} = 5 \\ 5x - \dfrac{2}{y} = 3 \end{array}\right\}$ (12) $\left.\begin{array}{l} \dfrac{x}{2} + \dfrac{y}{2} = 1 \\ \dfrac{x}{3} + \dfrac{y}{3} = 1 \end{array}\right\}$

(13) $\left.\begin{array}{l} \frac{1}{3}(x - y) = \frac{1}{4}(y - 1) \\ \frac{1}{7}(4x - 5y) = x - 7 \end{array}\right\}$

(14) $\left.\begin{array}{l} \dfrac{x}{14} + \dfrac{y}{18} = 1 \\ \dfrac{x + y}{2} + \dfrac{3x - 5y}{4} = 2 \end{array}\right\}$

III. Method of Elimination :

Already you have learnt to solve somultaneous equations by two methods, viz., the method of comparison and the method of substitution, where at first from both equations or one equation, one unknown quantity has been expressed in terms of the other unknown quantity and then equation involving one unknown has been formed.

Now observe how the two given equations can be reduced to a simple equation in one unknown in another way by the help of different mathematical operation.

Look at the following example :

Example 1.

Solve :

$$2x + 3y = 3 \quad \cdots \quad (i)$$

$$5x + 4y = 11 \quad \cdots \quad (ii)$$

Solution

Adding the terms onthe left hand hand sides and those of right hand sides of equations (i) and (ii), we get

$$7x + 7y = 14 \qquad \ldots \quad \text{(iii)}$$

Let us see what is the relation of equation (iii) with equations (i) and (ii). We express below, in the form of a table, the values of x from equations (*i*),(*ii*) and (*iii*) by substituting the values of y as –1, –2, –3, 1, 2, 3, in the above three equations.

Values of y	Values of x from equations (*i*)	Values of x from equations (*ii*)	Values of x from equations (*iii*)
1	0	$\frac{7}{5}$	1
2	$-\frac{3}{2}$	$\frac{3}{5}$	0
3	– 3	$-\frac{1}{5}$	–1
–1	3	3	3
–2	$\frac{9}{2}$	$\frac{19}{5}$	4
–3	6	$\frac{23}{5}$	5

Observe that when y = –1, in all the three cases the value of x remains the same i.e., x = 3, y = –1 are general solution of each of the equations (i), (ii) and (iii). That is all the solutions of the transformed equation obtained by adding the terms of the left hand sides and those of the right hand sides the two given equations are not the common solution of the given equations but one of the solutions (here x = 3, y = –1) is a general solution of the given simultaneous equations. This can be expressed in a slightly different way as: The general solution of the two equations is one of the solutions of the equation formed by adding the terms of left hand sides and those of right hand sides of the two equations. This argument is valid not only for addition but also for subtraction of the terms of the left hand sides and those of the right hand sides of the two given equations. For simplicity of our work some times we are to form a transformed equation from the given simultaneous equations in such a way that the transformed equation becomes a liniar equation involving one unknown quantity. We will see later how this can be done. Now the value of this unknown quantity and the value of the other unknown quantity (which may be found out from any one of the two given equations) will give the common solution of the simultaneous equations.

We know that if the terms of either side of an equation are multiplied on divided by the same number excepting zero, the new equation thus obtained has the same solution as that of the original equation.

Now observe that if terms of equation (i) is multiplied by 5 and those of equation (ii) by 2 then in both of them the coefficients of x become same.

Remark : for simplicity, instead of writing terms of equation (i), we write only equation (i) and so on

Multiplying equation (i) by 5, we have

$$10x + 15y = 15 \quad \ldots\ldots \quad \text{(iii)}$$

Also multiplying equation (ii) by 2, we have

$$10x + 8y = 22 \quad \ldots \quad \text{(iv)}$$

Subtracting equation (iv) from equation (iii), we have

$$7y = -7 \quad \ldots \quad \text{(v)}$$

Now let us see what is the relation of equations (i) and (ii) with equation (v). From equation (v) we have $y = -1$.

If we put this value of y in equations (i) and (ii), in all the cases we get the value of x equal to 3. So we find that $x = 3$, $y = -1$ satisfy both the equations. Therefore, the common solution of equations (i) and (ii) is $x = 3$, $y = -1$.

The steps for solving simultancious equations in this way are as follows :

(i) Observe whether the coefficients of any one of the unknowns in the two equations are same of not. If they are not same, then multiply the equations (or one equation if necessary) by two different numbers (or one number), so that in the transformed equations the coefficients of one of the unknowns become same.

(ii) Now add or subtract (in the case of same sign subtract and for different signs add) terms on both sides of the transfromed equations and remove that unknown quantity, so that the reduced equation contains only one unknown quantity.

(iii) From that equation find the value of that unknown quantity.

(iv) Substitute that value in any of the two equations and find the value of the other unknown quantity.

(v) These values of unknown quatities will be the common solution of the simultaneous equantions.

Observe the solution of the following equations with proper instruction :

Example 2. Solve :

$$5x - 2y = -4 \quad \ldots \quad (i)$$
$$-11x + 7y = 1 \quad \ldots \quad (ii)$$

Instructions	Method of solution
(a) Form the transformed equations by multiplying equation by such two smallest numbers such that the numerical value of the coefficients of x in the transformed equations becomes equal.	Multiply the two equations (i) by 11, we get $55x - 22y = -44$ ... (iii) Also multiplying equation (ii) by 5, we have $-55x + 35y = 5$ ... (iv)
Here if we multiply (*i*) by 11 and equation (*ii*) by 5, then the co-efficients of x (without considering their signs) in the two transformed equations will be same	
(*b*) Now add these two transformed equations (i.e. eliminate x).	Adding equations (*iii*) and (*iv*), we have $13y = -39$
(*c*) Find the value of y	$\therefore y = -\frac{39}{13} = -3$
(*d*) Subsitutute the	Substituting the

value of y (= –3) in equation (*i*) [or in equation (*ii*)] and and find the value of *x*	value of y in equation (*i*), we have $5x - 2(-3) = -4$ or, $5x = -10$ $\therefore x = -2$
(*e*) Now indicate the solution of the simultaneous equations	∴ the required solution is $\left.\begin{array}{l} x = -2 \\ y = -3 \end{array}\right\}$

Remarks :

(*i*) To transform two simultaneous equations to the equations having same coefficients of x, generally, the first should be multiplied by the coefficient of x of the seond and vice versa. To avoid multiplication by a large number, we multiply by such a number, so that the product becomes L.C.M. of the two co-efficients.

(*ii*) If in the transformed equations, the coefficients of the like terms have same sign, then one quantity is eliminated when one equation is subtracted from the other, If the signs are different, then the unknown quantity is eliminated when the equations are added.

Since in this method a pair of simultaneous equations are solved by eliminating one unknown it is called *method of elimination.*

Observe the solutions of the following equations by this method.

Example 3. Solve :

$$\frac{x}{3} - \frac{2}{y} = 1 \quad \cdots \quad (1)$$

$$\frac{x}{4} + \frac{3}{y} = 3 \quad \cdots \quad (2)$$

Soln. Multiplying equation (1) by 3 and equation (2) by 2, we have

$$x - \frac{6}{y} = 3$$

$$\frac{x}{2} + \frac{6}{y} = 6$$

Adding, we get $x + \frac{x}{2} = 9$ or, $\frac{3x}{2} = 9$

or, $3x = 18$ $\therefore x = \frac{18}{3} =$

6

Substituting 6 for x in equation (1), we have

$\frac{6}{3}-\frac{2}{y}=1$ or, $2-\frac{2}{y}=1$

or, $-\frac{2}{y}=-1 \therefore y=2$

Hence the required solution is

$$\left.\begin{array}{l} x=6 \\ y=2 \end{array}\right\}$$

Example 4. Solve : $ax+by=a^2$ ... (1)

$bx-ay=ab$ ... (2)

Soln. Multiplying equation (1) by a and equation (2) by b, we have

$$a^2x+aby=a^3$$
$$b^2x-aby=ab^2$$

Adding, we get $(a^2+b^2)x=a(a^2+b^2)$

or $x=\frac{a(a^2+b^2)}{a^2+b^2}=a$

Substituting a for x in equation (1), we have

$$a.\,a+by=a^2$$
$$\text{or, } a^2+by=a^2$$
$$\text{or, } by=a^2-a^2$$
$$\text{or, } by=0$$
$$\text{or, } y=\frac{0}{b}=0$$

Hence the required solution is

$$\left.\begin{array}{l} x=a \\ y=0 \end{array}\right\}$$

Exercise 12

I. If x + y = 4, complete the following table

x								
y	1	2	3	4	–1	–2	–3	–4

II. If $x-2y=1$, then make a similar table and fill up the corresponding values of x.

III. Add the above two equations. Make a table as above for the new transformed equation.

IV. From these three table write down the common values of x and y.

V. For two equations $3x + 4y = 24$ and $2x + 3y = 17$ make two tables as above. Is it possible to solve the simultaneous equations from these two tables ?

VI. Write down the transformed equations formed by adding and subtracting the two equations in question III. Make two tables as above. What are the two common values of x and y in the tables ? With those common values be the general solution of the equations of question III ?

Exercise 13

Solve the following equations by the method of elimination :

(1) $\left.\begin{aligned} 5x - 3y &= 9 \\ 2x + 5y &= 16 \end{aligned}\right\}$ (2) $\left.\begin{aligned} x + y &= 3 \\ 4x - 5y &= -6 \end{aligned}\right\}$

(3) $\left.\begin{aligned} 2x + 3y &= 32 \\ 11y - 9x &= 3 \end{aligned}\right\}$ (4) $\left.\begin{aligned} x + y &= 48 \\ x + 4 &= \frac{5}{2}(y + 4) \end{aligned}\right\}$

(5) $\left.\begin{aligned} \frac{x}{2} + \frac{y}{3} &= 8 \\ \frac{5x}{4} - 3y &= -3 \end{aligned}\right\}$ (6) $\left.\begin{aligned} 13x - 12y &= -15 \\ 8x - 7y &= 0 \end{aligned}\right\}$

(7) $\left.\begin{aligned} 3x - \frac{2}{y} &= 5 \\ x + \frac{4}{y} &= 4 \end{aligned}\right\}$ (8) $\left.\begin{aligned} 8x + 5y - 11 &= 0 \\ 3x - 4y - 10 &= 0 \end{aligned}\right\}$

(9) $\left.\begin{aligned} \frac{x}{3} + \frac{y}{4} &= 0 \\ 3x - 7y &= 37 \end{aligned}\right\}$ (10) $\left.\begin{aligned} \frac{x}{3} &= \frac{y}{4} \\ 7x - 5y &= 2 \end{aligned}\right\}$

(11) $\left.\begin{aligned} 3x - 4y &= 1 \\ 4x - 3y &= 6 \end{aligned}\right\}$ (12) $\left.\begin{aligned} 2x + 3y - 7 &= 0 \\ 3x + 2y - 8 &= 0 \end{aligned}\right\}$

(13) $\left.\begin{aligned} \frac{x + y}{2} + \frac{3x - 5y}{4} &= 2 \\ \frac{x}{14} + \frac{y}{18} &= 1 \end{aligned}\right\}$ (14) $\left.\begin{aligned} 2x + 3y &= 5 \\ 5x - 4y &= -22 \end{aligned}\right\}$

(15) $\left.\begin{aligned} \frac{x}{2} + \frac{y}{3} &= 1 \\ \frac{x}{3} + \frac{y}{2} &= 1 \end{aligned}\right\}$ (16) $\left.\begin{aligned} 2x + \frac{3}{y} &= 1 \\ 5x + \frac{2}{y} &= \frac{11}{12} \end{aligned}\right\}$

(17) $\left.\begin{array}{l}4x - 3y = 16\\ 6x + 5y = 62\end{array}\right\}$ (18) $\left.\begin{array}{l}2(x - y) = 3\\ 5x - 8y = 14\end{array}\right\}$

(19) $\left.\begin{array}{l}ax + by = c\\ a^2x + b^2y = c^2\end{array}\right\}$ (20) $\left.\begin{array}{l}x + y = a + b\\ ax - by = a^2 - b^2\end{array}\right\}$

IV. Method of Cross-multiplication

The general form of a linear equation in two unknowns is $ax + by + c = 0 (a, b \neq 0)$

Two such simultaneous equations may be written as

$$a_1x + b_1y + c_1 = 0 \quad \dots \quad (i)$$

$$a_2x + b_2y + c_2 = 0 \quad \dots \quad (ii)$$

Let us solve them by the method of elimination. Multiplying equation (*i*) by a_2 and equation (*ii*) by a_1, we have

$$a_1a_2x + a_2b_1y + a_2c_1 = 0$$

$$\underline{a_1a_2x + a_1b_2y + a_1c_2 = 0}$$

Subtracting, we have

$$a_2b_1y - a_1b_2y + a_2c_1 - a_1c_2 = 0$$

$$\text{or, } y(b_1a_2 - b_2a_1) = c_2a_1 - c_1a_2$$

$$\therefore\ y = \frac{c_2a_1 - c_1a_2}{b_1a_2 - b_2a_1} = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

$$(\because\ a_1b_2 - a_2b_1 \neq 0)$$

$$\frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1} \quad \dots \quad (iii)$$

Again multiplying equations (*i*) and (*ii*) respectively by b_2 and b_1 we have

$$a_1b_2x + b_1b_2y + b_2c_1 = 0$$

$$\underline{a_2b_1x + b_1b_2y + b_1c_2 = 0}$$

Subtracting, we have

$$a_1b_2x - b_2b_1x + b_2c_1 - b_1c_2 = 0$$

$$\text{or, } x(a_1b_2 - a_2b_1) = b_1c_2 - b_2c_1$$

$$\therefore \quad x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \quad (\because a_1b_2 - a_2b_1 \neq 0)$$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{1}{a_1b_2 - a_2b_1} \quad \cdots \quad (iv)$$

From equation (iii) and (iv), we have

$$\frac{x}{b_1c_2 - c_2a_1} = \frac{y}{c_1a_2 - b_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

From this relation we come to know the relation between the values x and y of the simultaneous equations with those of the coefficients of x and y and the constant terms. We may take this relation as a formula and use this to solve any two simultaneous equations. Without following the steps of the method of elimination we can straightway find the solution of the equations.

The formula for the cross-multiplication and from that the method of solution of two simultaneous equations can be expressed as follows :

From the simultaneous equations

$$a_1x + b_1y + c_1 = 0 \quad \cdots \quad (i)$$

$$\text{and } a_2x + b_2y + c_2 = 0 \quad \cdots \quad (ii)$$

We get cross multiplication

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1} \quad \cdots \quad (A)$$

(where $a_1b_2 - a_2b_1 \neq 0$)

i.e., $x = \dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

So it is clear that the most important thing is to write the relation (A) by cross-multiplication

First write down the coefficients indifendent per of the two equations in the following form :

$$\begin{array}{cccc} b_1 & c_1 & a_1 & b_1 \\ b_2 & c_2 & a_2 & b_2 \end{array}$$

Fig. 1

Now multiply the coefficients according to the arrow heads and subtract the upward product from the downward product. In this way you will get three diffferences. Place these differences under x, y and 1 respectively and connect them by the sign of equality.

Example 1. Solve :

$$8x + 5y = 11$$
$$3x - 4y = 10$$

Soln.

By transposition, we get

$$8x + 5y - 11 = 0$$
$$3x - 4y - 10 = 0$$

By cross-multiplication, we get

$$\frac{x}{5(-10) - (-4)(-11)} = \frac{y}{(-11)3 - (-10)8} = \frac{1}{8(-4) - 3.5}$$

or, $\dfrac{x}{-50 - 44} = \dfrac{y}{-33 + 80} = \dfrac{1}{-32 - 15}$

or, $\dfrac{x}{-94} = \dfrac{y}{47} = \dfrac{1}{-47}$

or, $x = \dfrac{-94}{-47} = 2$ and $y = \dfrac{47}{-47} = -1$

$\therefore$ the required solution is $\left.\begin{array}{l} x = 2 \\ y = -1 \end{array}\right\}$

Example 2. Solve :

$$3x + 4y - 17 = 0 = 4x - 3y - 6$$

Soln. $3x + 4y - 17 = 0$

$$4x - 3y - 6 = 0$$

By cross multiplication, we get

$$\frac{x}{4(-6) - (-3)(-17)} = \frac{y}{(-17)4 - (-6)3} = \frac{1}{3(-3) - 4.4}$$

or, $\dfrac{x}{-24 - 51} = \dfrac{y}{-68 + 18} = \dfrac{1}{-9 - 16}$

or, $\dfrac{x}{-75} = \dfrac{y}{-50} = \dfrac{1}{-25}$ or $\dfrac{x}{3} = \dfrac{y}{2} = 1$ (multiplying by -25)

or, $x = 3$ and $y = 2$

$\therefore$ the required solution is $\left.\begin{aligned} x &= 3 \\ y &= 2 \end{aligned}\right\}$

Example 3. Solve :

$$ax + by - c^2 = 0$$
$$a^2x + b^2y - c^2 = 0$$

Solv. By cross multiplication, we get

$$\frac{x}{b(-c^2) - b^2(-c^2)} = \frac{y}{(-c^2)a^2 - (-c^2)a} = \frac{c^2}{ab^2 - a^2}$$

$$\text{or,} \quad \frac{x}{-b + b^2} = \frac{y}{-a^2 + a} = \frac{c^2}{ab^2 - a^2b}$$

$$\text{or,} \quad \frac{x}{bc^2 + b^2c^2} = \frac{y}{-c^2a^2 + c^2a} = \frac{1}{ab^2 - a^2b}$$

$$\text{or,} \quad \frac{x}{b(1-b)} = \frac{y}{a(a-1)} = \frac{c^2}{ab(a-b)}$$

$$\text{or,} \quad x = \frac{bc^2(1-b)}{ab(a-b)} = \frac{c^2(1-b)}{ab(a-b)}$$

$$\text{or,} \quad y = \frac{C62a(a-1)}{ab(a-b)} = \frac{c^2(1-b)}{ab(a-b)}$$

Hence the required solution is

$$\left.\begin{aligned} x &= \frac{c^2(1-b)}{a(a-b)} \\ y &= \frac{c^2(1-b)}{a(a-b)} \end{aligned}\right\}$$

Remarks : If the term independent of x and y, that is if the constant term is same (as c^2 in this case), then instead of 1 that constant can be written in the numerator of the third factor.

Exercise 14

Solve by the method of cross-multiplication :

(1) $\left.\begin{aligned} 2x + 3y &= 7 \\ 5x - 2y &= 8 \end{aligned}\right\}$ (2) $\left.\begin{aligned} 5x + 3y &= 11 \\ 2x - 7y &= -12 \end{aligned}\right\}$

$$(3)\ \left.\begin{array}{l}6x+20y=7\\3x-8y=8\end{array}\right\}\qquad(4)\ \left.\begin{array}{l}3x+4y-5=0\\5x+12y-3=0\end{array}\right\}$$

$$(5)\ \left.\begin{array}{l}3x+4y-17=0\\4x-3y-6=0\end{array}\right\}\qquad(6)\ \left.\begin{array}{l}7x-3y-31=0\\9x-5y-41=0\end{array}\right\}$$

$$(7)\ \frac{x}{6}-\frac{y}{3}=\frac{x}{12}-\frac{2y}{3}=4\qquad(8)\ \frac{x}{5}+\frac{y}{3}=\frac{x}{4}-\frac{y}{3}-\frac{3}{20}=0$$

$$(9)\ \left.\begin{array}{l}\frac{x+2}{7}+\frac{y-x}{4}=2x-8\\\frac{2y-3x}{3}+2y=3x+4\end{array}\right\}\qquad(10)\ \left.\begin{array}{l}x+y-3=0\\4x-5y-6=0\end{array}\right\}$$

$$(11)\ \left.\begin{array}{l}x+5y=36\\\frac{x+y}{x-y}=\frac{5}{3}\end{array}\right\}\qquad(12)\ \left.\begin{array}{l}x-y=2a\\ax+by=a^2+b^2\end{array}\right\}$$

$$(13)\ \left.\begin{array}{l}x+y=2b\\x-y=2a\end{array}\right\}\qquad(14)\ \left.\begin{array}{l}13x-12y+15=0\\8x-7y=0\end{array}\right\}$$

$$(15)\ \left.\begin{array}{l}3x-4y=1\\x=3y+6\end{array}\right\}\qquad(16)\ \left.\begin{array}{l}ax+by=c\\a^2x+b^2y=c^2\end{array}\right\}$$

$$(17)\ \left.\begin{array}{l}\frac{x}{a}+\frac{y}{b}=2\\ax-by=a^2-b^2\end{array}\right\}\qquad(18)\ \left.\begin{array}{l}ax+by=c\\bx+ay=1+c\end{array}\right\}$$

$$(19)\ \left.\begin{array}{l}ax+by=1\\bx+ay=\frac{2ab}{a^2+b^2}\end{array}\right\}\qquad(20)\ \left.\begin{array}{l}bx-ay=0\\ax+by=a^2+b^2\end{array}\right\}$$

[Remarks : Although some equations given in this chapter are not linear in true sense, they have been considered for their importance and as they are reducible to linear equation.]

CHAPTER 4

Solution of different problems with the help of simple simultaneous equations

You have already learnt how different types of problems can be solved with the help of simple equations. Taking the value of the unknowns as x,an equation is formed from the given condition of the problem. Solving that equation the value of x or the unknown quantity can be found out.

In many cases it is required to determine the values of two different unknown quantites in a problem. In that case, the values of two different unknown qnantities are taken as x and y. Then from the given condition or conditions of the problem one or more equations can be formed. On solving these equations the values of x and y can be determined.

Let us consider the following problem :

Example 1 The cost of 3 cows and 5 goats is Rs. 3650/-. Again the cost of 7 cows and goats is Rs. 6610/-. What is the cost of a cow and that of a goat.

Solution : Here the unknown quantities are the cost of a cow and that of a goat.

Let the cost of a cow be Rs. x and that of a goat be Rs. y.

Observe that in the given problem there are two given conditions. Form the first condition, we have

$$3x + 5y = 3650 \quad \cdots \quad \text{(i)}$$

and from the second condition, we have

$$7x + 3y = 6610 \quad \cdots \quad \text{(ii)}$$

Multiplying Eqn. (i) by 3 and Eqn. (ii), by 5 we have

$$9x + 15y = 10950 \ldots \quad \text{(iii)}$$

$$35x + 15y = 33050 \quad \cdots \quad \text{(iv)}$$

Subtracting Eqn. (iii) from Eqn. (iv) we have,

$$26x = 22100$$

$$\therefore\ x = \frac{22100}{26} = 850$$

Substituting this value of x in Eqn. (i), we have

$$3 \times 850 + 5y = 3650$$

$$\therefore\ 5y = 3650 - 2550 = 1100$$

$$\therefore\ y = 220$$

$\therefore$ the cost of a cow = Rs. 850 and the cost of a goat = Rs. 220

Remark : Taking the cost of a cow as Rs. 850 and that of a goat as Rs. 220 verify that the two given conditions of the problem are satisfied. In this way solving every problem by the help of algebra, always verify whether the given conditions are satisfied or not. If they are saitisfied, the solution is correct.

Example 2. Selling rice and wheat two persons received a total sum of Rs. 2500. The first person got 30% of the sale proceeds of rice and 70% of that of wheat. His share was Rs. 1350. If the rest amout be received by the second person, what is the total amount of money did the second person obtain from selling wheat.

Solution : Let the total amount of money obtained by selling rice by Rs. x and the total amount of money obtained by selling wheat by Rs. y. Then from the given conditions, we have

$$x + y = 2500 \qquad \ldots \quad (i)$$

$$\text{and } \frac{3}{10}x + \frac{7}{10}y = 1350 \qquad \ldots \quad (ii)$$

Solving eqns. (i) and (ii) simultaneously, we have

$$x = 1000 \text{ and } y = 1500$$

$\therefore$ the amount of money obtained by selling rice = Rs. 1000. By selling wheat the second person received 30% of Rs. 1500 i.e., Rs. 450.

Example 3. Ashish babu bought some mangoes at the rate of 15 rupees a dozen and some oranges at the rate of 6 rupees a dozen and found that those together cost Rs. 35. It would have cost him Rs. 85 had he bought double the number of mangoes and three times the number of oranges. How many mangoes and how many organges did he buy ?

Solution. Let us suppose that he bought x number of mangoes and y number of oranges.

The price of x mangoes $= \text{Rs. } x.\frac{5}{4} = \text{Rs. } \frac{5x}{4}$

and the price of y oranges $= \text{Rs. } y.\ \frac{1}{2} = \text{Rs. } \frac{y}{2}$

From the first condition of the problem, we have

$$\frac{5x}{4} + \frac{y}{2} = 35 \qquad \dots \qquad \text{(i)}$$

and from the second condition of the problem, we have

$$\frac{5x}{2} + \frac{3y}{2} = 85 \qquad \dots \qquad \text{(ii)}$$

Solving them we get $x = 16$ and $y = 20$

$\therefore$ he bought 16 mangoes and 20 oranges.

Example 4. A number consisting of two digits exceeds four times the sum of its digits by 3. If the places of the digits are interchanged, the new number thus formed exceeds the original number by 18. Find the number.

Solution. Let y be the digit in the unit's place and x that in the ten's place. So the number is

$$10x + y$$

The sum of the digits of this number is $x + y$.

From the first condition, we have

$$10x + y - 4(x + y) = 3$$

i.e., $6x - 3y = 3$

or $2x - y = 1 \qquad \dots \qquad (i)$

Again, if the digits in the unit's and ten's places be interchanged, the new number thus formed is $10y + x$.

From the second condition, we have

$$10y + x - (10x + y) = 18$$

or, $9y - 9x = 18$

or, $y - x = 2 \qquad \dots \qquad (ii)$

Solving (*i*) and (*ii*), we have $x = 3, y = 5$

Thus the required number is 35.

Example 5. Sum of the ages of father and his son is 55 years. After 16 years father's age becomes double that of his son. What are their present ages ?

Solution. Let x *and* y be the ages of the father and his son respectively.

Them from the first condition, we have

$$x + y = 55 \qquad \ldots \qquad (i)$$

Again, from the second condition of the problem, we have,

$$x + 16 = 2(y + 16)$$

$$\text{i.e., } x - 2y = 16 \qquad \ldots \qquad (ii)$$

Solving (*i*) and (*ii*), we have $x = 42$ and $y = 13$.

$\therefore$ the ages of the father and the son and 42 years and 13 yeas respectively.

[*Remark* : If x be the age of the father, then the age of the son is $55 - x$ and from the second condition of the problem $x + 16 = 2(55 - x + 16)$, which is a simple linear equation in x.

Solving this we can find out the ages if the father and te son.]

Example 6 Two trains start at the same time from Radhapur and Bishnupur towards each other. The two place are 650 km. apart and te trains cross each other afte 10 hours. Had the first train started 4 hours 20 minutes after 8 hours. Find the speeds of the trains.

Solution. Let the speed of the first train be x km. per hour and that of the second be y km. per hour.

Now from the given condition of the problem, we have

$$10x + 10y = 650$$

$$\text{i.e., } x + y = 65 \qquad \ldots \qquad (i)$$

Again, from the second condition, we have

$$8x + 12\tfrac{1}{3}y = 650$$

$$\text{i.e., } 24x + 37y = 1950 \qquad \ldots \qquad (ii)$$

Solving (*i*) and (*ii*), we have$x = 30$ and $y = 35$

∴ the speed of the first train is 30 km. per hour and that of the second train is 35 km. per hour.

Exercise 15

1. The cost of two tables and three chairs together is Rs. 1075. Again the cost of 3 tables and 8 chars together is Rs. 1875. What are the respective costs of a chair and of a table ?

2. The cost of 5 pens and 3 penciles together is Rs. 17.40 and that of 7 pens and 6 pencils together is Rs. 25.80. Find the price of a pen and that of a pencil.

3. Given that $s = ut + \frac{1}{2} ft^2$, $s = 21$ when $t = 3$ and $s = 45$ when $t = 5$, what will be the values of u and f ?

4. $\frac{2}{3}$ of a work is completed when Dben works for 4 days and Ramen for 3 days. Again if Deben works for 3 days and Ramen for 6 days $\frac{11}{12}$ of the work is done, How many days would Deben and Ramen take separately to complete the work ?

5. Sum of the weights of Pradip and Prakash together is 85 kg. Also half of Pradip's weight is equal to $\frac{4}{9}$ of Prakash's weight. Find the weights of Pradip and Prakash.

6. The age of Subash Babu is twice that of his son. 10 years back his age was three times him son's age. Find their present ages.

7. From a bank Deb Kumar babu encashed a cheque for Rs. 590. he received altogether 70 notes and of denorminations Rs. 5 and Rs. 10. Find the number of 5-rupee notes and 10-rupee notes.

8. In two solutions of 100 gms. each the first one contains 5 gms. of sugar and the second one contains 10 gms. of suger. Taking 150 gms. from these two solutions it is found to contain $9\frac{2}{3}$ gms. of sugar. What are the weight of the first and second solutios in that 150 gms. of solution ?

9. Sutapa.Devi deposited a part of her money at the rate of $5\frac{1}{2}\%$ and another part at the rate of $5\frac{1}{4}\%$ in a bank. Her annual income from these two savings if Rs. 505. It is found that the amount of

interest obtained from teh first saving is Rs. 45 more than that obtained from the second. Find the amounts deposited in the first and the second parts of her savings.

10. In two number if 21 is added to the first it becomes double of the second. Again when 4 is added to the second, it becomes double of the first. Find the numbers.

11. To a fraction if 2 is added to both the numerator and te denominator it becomes $\frac{7}{9}$ and it becomes $\frac{1}{2}$ when 3 is subtracted from both the numerator and the denominator. Find the fraction.

12. The cost of 3 kg. of mustard oil and 2 kg of coconut oil together is Rs. 104. Again 5 kg of mustard oil and 3 kg of coconut oil together cost Rs. 164. Find the costs of a kg of mustard oil and a kg of coconut oil separately.

13. Akhil Banu and Amar Babu were both candidates in a village Panchayat Election. Akhil Babu defeated Amar Babu by a margin of 75 votes. had 20% of the voters who voted for Akhil Babu voted for Amar Babu, then the latter would hae won by 19 votes. What are the number of votes polled by each ?

14. Latif told Ishan, "If you give $\frac{1}{3}$ of your money to me, mine becomes Rs. 200'. Ishan told Latif, "If you give $\frac{1}{2}$ your money to me then mine will also be Rs. 200". Find the amount than the original one. Find the number.

15. A number consisting of two digist is such that its digit in the ten's place is twice that in the unit's place. If the digits are reversed, the number thus formed is 27 less than the original one. Find the number.

16. The sun of the digits of a number consisting of two digits is 8. If 18 is added to the number, the digits of the number are interchanged. Find the number.

17. If the length and the breadth of a rectangular floor are increased by 2 m. and 3 m. respectively, its area would be 75 sq. m. more. If the length is decreased by 2 m. and the breadth is increased by 3 m., the area would increase by 15 sq. m. Find the length and the breadth of the floor.

18. Along the current a man covered a distance of 150 miles in 5 hours and returned against the current in $7\frac{1}{2}$ hours. What are the respective speeds of the both and of the current ?

19. The cost of 7 metres of terry-cotton and 20 metres of cotton clothes cost together Rs. 460. Find the cost per metre of terry-cotton and cotton clothes

20. When your age was same as my present age then my age was half of your present age. The sum of our present ages is 63 years. What are our respective ages ?

21. 5 times the bigger number is equal to 8 times the smaller and twice the smaller number is equal to 10 more than the bigger number. What ar the numbers ?

22. Mazid bought a cycle and and a watch together for Rs. 1700. he sold the cycle at a profit of 15% and the watch at a loss of 20% and found that he has made a net profit of Rs. 80. Find the cost price of the cycle and of the watch.

23. A sum of money is distributed among some flood affected boys. If there were 5 boys less, then each would have received Rs. 3 and if there were three boys more, then each would have received Rs. 1.50 what sum of money was distribured and what was the number of boys.

24. On keeping his orange in sum boxes and orange seller found that if he keeps 20 oranges more in each box then it requires three boxes less and it he keeps 5 oranges less in each box he requires on more box. Find how many oranges he had and how many boxes ?

25. Two workers complete a piece of work in 12 hours. If the first worker did half the work and the second did the remaining half then it would have take 25 hours to complete he work. Find the number of hours required by each worker to complete the work alone.

26. A bag contained a sum of Rs. 260 in 1 rupee and 50 p coins. If half of the 1 rupee coins is replaced by same number of 50 p coins them the total sum in the bag becomes Rs. 215. Find how many 1 rupee coins and 50 p coins were there at first in the bag.

27. A motor car travelled a distance with a uniform speed. If the speed of the car would have been 7 km per hour more, then it would have taken 3 hours less to cover the above distance and if the speed would have been 6 km per hour less, then it would have taken 3 hours more to cover the same distance. Calculate the distance travelled by the car and the speed of the car.

28. To travel on a boat 30 km against the current and 44 km along the current a man takes 10 hours. If he travels 40 km against the current and 55 km along, it takes 13 hours. Find the speed of the boat and that of the current.

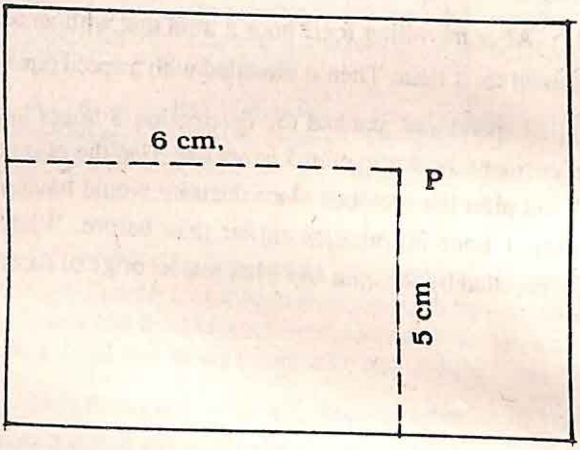
29. A man buys two kinds of bananas at the rate of 8 and 4 bananas a rupedd respectively. He loses Rs. 1.20 on selling each banana at the rate of 14 p. Had he sold those ar the rate of 6 bananas a rupee he would have gained Rs. 2. How many bananas of each kind did he buy ?

30. After travelling for 1 hour a train met with an accident and was detained for 1 hour. Then it travelled with a speed equal to $\frac{3}{5}$th of the original speed and reached the destination 3 hours late. Had the place of accident the destination 3 hours late. Had the place of accident been 50 km after the previous place the train would have reached the destination 1 hour 20 minutes earlier than before. What was total distance travelled by the train and what was its original speed ?

CHAPTER 5

Graphs (Two dimensional)

You know that there are infinite number of points on a plane. In many cases it is necessary to know the exact position of a point on the plane. Suppose you have marked a point in your exercise book by a pencil. Let that point be P. How will you explain the position of P ?



From any side of your book you can measure the distance of that point by a scale. Let the distance of the point P from the bottom of the book be 5 cm. Then we can say that the point P is 5 cm above the bottom side of the book. If you think for a little you can easily find that it is not possible to give the exact location of P in this way. Since there are many more points which are are all 5 cm above the bottom of the book. Now measure the distance of P from the left side of the book. Let this distance be 6 cm. So the point P is 6 cm away from the left side of the book. But as before we cannot exactly locate the point P, since from the left side there are infinitely many points which are

all 6 cm away from that side. But if we say that the point *P* is at a distance of 6 cm from the left side and at a distance of 5 cm from the bottom side of the book, then *P* can be exactly located.

The reason being that there is only one point which is at a distance of 6 cm from the left side and at the same time 5 cm from the bottom side of the book. So if the distance of the point *P* from these two sides of the book are known we can locate it exactly.

You can easily understand from the above discussion that the location of a point on a plane can be exactly given by the help of its distances from two intersecting straight lines on the plane. Each of these two inrtersecting lines is called axis (plural - axes). These two axes can be drwan through any point on the plane. Generally two mutually prependicular straight lines are taken as the axes. Of course, there are many system of axes which are inclined at any angle. In our discussion we shall take axes as lines mutually perpendicular.

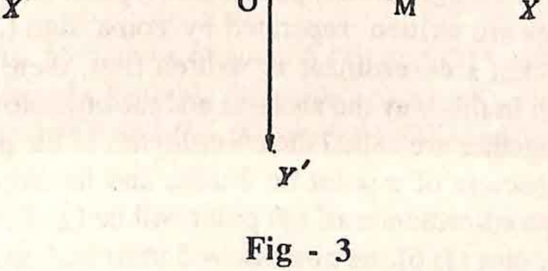


Fig - 3

Let us suppose that the axes XOX' and YOY' intersect each other at the point O. The point O is called the origin. The axes *XOX'* and *YOY'* are respectively called the *x*-axis and the y-axis. It is convenient to take the *x*-axis horizontally and the *y*-axis vertically.

Suppose that we have to determine the location of a point P on that plane. From the point *P*, two perpendiculars *PM* and *PN* are drawn on *x* and *y*-axes respectively. Then with the help of these two perpendicular distances the location of *P* will be determined. These two perpendicular distances are called the co-ordinates of *P*. Among the two perpendicular distances, mentioned above, the distance which is parallel to the *x*-axis is called the abscissa of the point *P* and the distance which is parallel to the *y*-axis is called the ordinate of the point *P*.

In the figure, $PN = OM$ = abscissa of the point *P*
and $PM = ON$ = ordinate of the point *P*

Generally, we denote the abscissa by the symbol *x* and ordiante by the symbol *y*. That is why the abscissa is called the *x* co-ordinate and the ordinate is called the *y* co-ordinate.

Thus it is found that every point on a plane is associated with two co-ordinates, one is the *x* co-ordinate or the abscissa and the other is the *y* co-ordinate or the ordinate of the point. To explain the position of a point these two co-ordinates are written separated by 'coma' sign (,). Within the bracket *x* co-ordinate is written first, then the *y* co-ordinate. In this way the abscissa and the ordinate of a point taken together are called the co-ordinates of the point, e.g., if the abscissa of a point be 2 units and its ordinate be 3 units, the co-ordinates of the point will be (2, 3). Similarly for the point (5, 6) the abscissa = 5 units and ordinate = 6 units.

Remarks :

(1) The co-ordinates of the origin are (0, 0).

(2) By a little observation it can be easily seen that *y* co-ordinate of any point on the *x*-axis is zero and *x* co-ordinate of any point on the y-axis is zero.

(3) Taking any point on the plane as the origin two mutually perpendicular lines through this point can be taken as the *x* and the *y*-axes (y) shift here the portion (ii). Therefore, with different points as orgin, the same point will have different co-ordinates.

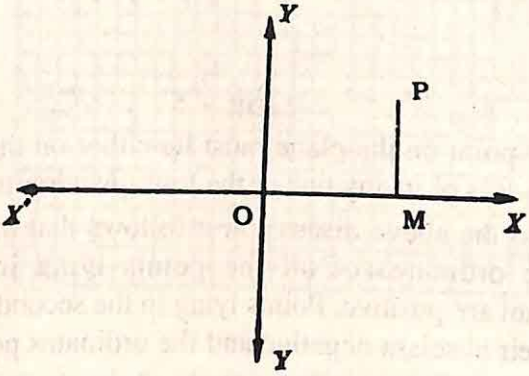


Fig - 4

In your exercise book through any point *O* draw the two mutually perpendicular axes *XOX'* and *YOY'*. The origin *O* has divided the *x*-axes into two parts *OX* and *OX'*. So the values of *x* will have different signs along *OX* and *OX'*.

Similarly, the values of *y* along *OY* and *OY'* will have different signs. In general, the values of *x* and *y* are both positive along *OX* and *OY*. Then along *OX'* and *OY'* the values of *x* and *y* will be negagive.

Observe that the *x*-axis and the *y*-axis have divided the plane into four parts viz *XOY*, *YOX'*, *X'OY'* and *Y'OX*. These four parts are called four quadrants.

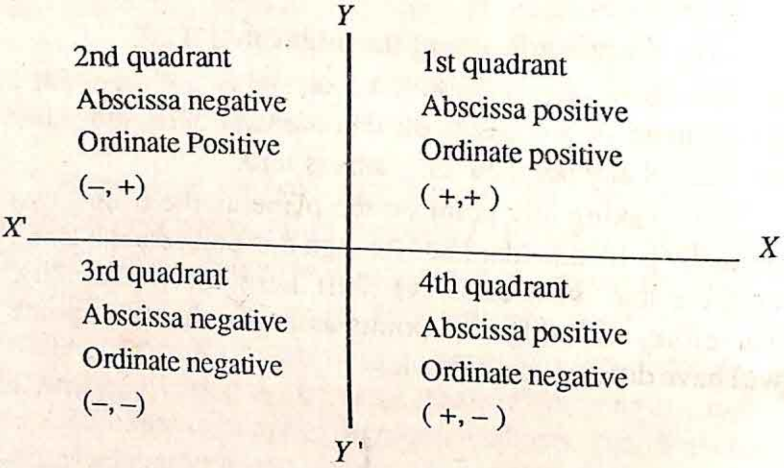


Fig - 5

Any point on the plane must lie either on the *x*-axis or on the *y*-axis or in any one of the four quandrants.

From the above discussion it follows that the abscissa and the ordinates of all the points lying in the first quandrant are positive. Points lying in the second quandrant have their abscissa negative and the ordinates positive; the points in the third quadrants have their abscissa and the ordinates both negative and all points lying in the fourth quanrant have their abscissa positive and the ordinates negative.

So we find that

(i) the points (1, 5), (2, 7), (15, 12) etc. lie in the first quandrant ;

(ii) the points (–2, 3), (–10, 12), (–6, 5) etc. lie in the second quandrant.

(iii) the points (–2, –3), (–5, –8), (–7, –12), etc. lie in the third quandrant ;

(iv) the points (2, –8), (5, –6), (4,–12), etc. lie in the fourth quandrant.

Graph paper :

To express the location of a point on a plane a squared paper is used. The paper is divided into small squares by drawing horizontal lines and vertical lines which are equidistant from each other. This type of squared paper is called graph paper. One unit may be taken as the length of sides of one or more of the smallest squares.

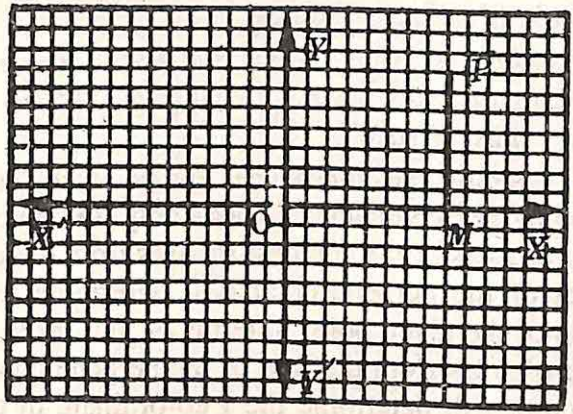


Fig - 6

Determination of co-ordinates of a point lying on a graph paper :

Generally, there are deep thick lines after the smallest squares in the graph paper. For simplicity of our work x-axis is taken along any one of such horizontal thick line and y-axis along a vertical thick line. The names of the axes are given as XOX' and YOY'

Now perpendiculars are to be drawn to the x-axis and to the y-axis from the point whose co-ordinates are to be found.

The number of sides of the smallest squares from the origin O to the foot of the perpendicular to x-axis, are counted. If the length of one side of the smallest square is taken as unit, then the number of sides of those smallest squares will be equal to the abscissa of the point.

In a similar way the number of sides of the smallest square from origin to the foot of the perpendicular to the y-axis are counted. Like x-axis if one side of the smallest square along y-axis is taken as unit, then the above number will give the ordinate of the point. The abscissa and ordinate will be positive or negative according to the quadrant in which the point lies.

Remarks : (i) Along x-axis and y-axis, one or more sides of smallest squares may be taken as one unit, but the unit taken must be stated.

(4) Generally, the co-ordinates of a point are determined by drawing perpendicular on x-axis only. In that case, the number of sides of the smallest squares from the origin to the foot of the perpendicular will determine the x co-ordinate of the given point in terms of unit taken and the number of sides from the foot of the perpendicular to the given point will determine the y co-ordinate in terms of corresponding unit.

Method of plotting a point on a graph paper :

If we know the co-ordinates of a point, then we can plot it on the graph paper in the following way.

With the help of a scale we draw one horizontal line XOX' and a vertical line YOY' on the graph paper and take them respectively as x and y-axes. Then we have to decide the number of sides of the smallest squares the length of which will be taken as unit. According to this unit and the signs of abscissa and ordinate the location of the point is found.

The example below will clarify the process.

Example 1 : Plot the following points on a graph paper :
A $(2, 4)$, B $(-3, -2)$, C $(-5, 7)$ *and* D $(5, -10)$

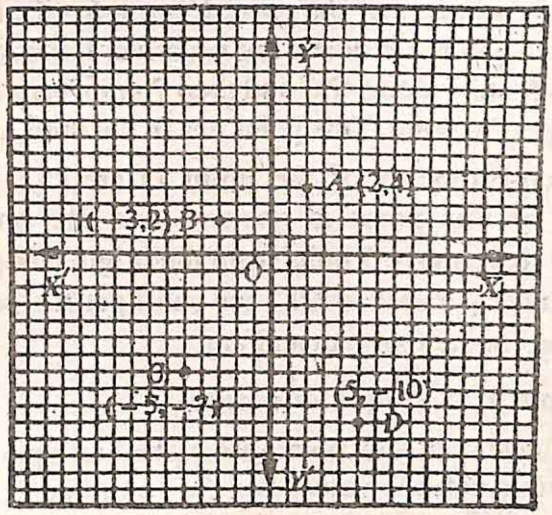


Fig - 7

On the graph paper XOX' and YOY' are drawn as the two axes. The length of one side of the smallest square is taken as unit.

The abscissa and the ordinate of the point A(2,4) are both positive, i.e., the point A lies in the first quadrant. Along OX one has to move 2 units from the origin O (i.e. wide of two smallest squares) and then from that point 4 units (i.e side of four smallest squares) parallel to OY. Thus we get the point. This point may be marked by a pencil and the co-ordinates (2, 4) of the point are to be written.

The co-ordinates of the point B are (–3, 2). The abscissa of the point is negative and the ordinate is positive. If we move from the origin 3 units along OX' and then 2 units from there parallel to OY, we shall get the position of the point. This point is marked by a pencil and the co-ordinates. (–3, 2) are written by its side. This point lies in the second quadrant.

Solution. The values of x and y for which the given equation is satisfied can be found out by the following procedure.

$$4x + 3y = 12$$

$$\text{Or,}\quad 3y = 12 - 4x$$

$$\text{Or,}\quad y = \frac{12-4x}{3}$$

From the above equation the value of y can be easily obtained corresponding to any value of x.

For, when $x = 0, y = \frac{12-4\times 0}{3} = 4$

Similarly, when $x = 3, y = 0$

When $x = 6,\ y = -4$

when $x = -3,\ y = 8$ etc.

The values of x and y can be written in the tabular form as given below :

x	0	3	6	–3
y	4	0	–4	8

In this way infinite number of values of x and y can be determined, which will satisfy the given equation.

In general three pairs of corresponding values of x and y are taken. For plotting the values of x and y on the graph paper it is convenient not to take fractional values of x and y.

Next, through any point O on the graph paper two mutually perpendicular axes XOX' and YOY' are drawn. Taking the length of one or more sides of the smallest square as unit (generally length of one side is taken as unit) the points (0, 4), (3, 0), (6,–4) and (–3, 8) are plotted on the graph paper. By the side of every point their corresponding co-ordinates are to be written. Then the straight line, obtained by joining these points and extending it in both directions, will represent the graph of the given equation.

It should be remembered that the required graph of the equation is not the line segment obtained by plotting the solution points but the straight line obtained by its infinite extension in both directions. (Fig. No. 9)

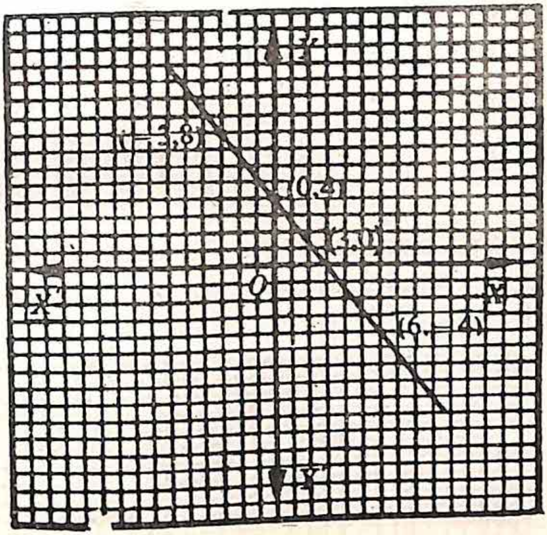


Fig - 9

(Graph of the equation $4x + 3y = 12$)

Remark : If the x and y co-ordinates of a point are both fractions, then along x and y axes suitable units may be taken to plot that point.

Take any point on the graph of the equation and find its corresponding co-ordinates. In the given equation substitute for x and y the corresponding values of the x and y co-ordinates.

What do you observe ?

The co-ordinaties of any point on the graph satisfy the original equation. Again for any solution of the original equation there is a point on the graph.

Thus we can say that the geometrical representation of a simple equation is a straight line and the algebraic representation of a straight line is a simple equation in one or two unknowns.

Drawing the graph of a linear equation with one unknown quantity.

Suppose we have to draw the graph of the equation $x=5$. This equation may be written as $x - 0.y = 5$

Here observe that for any value of y, the value of x is always equal to 5. So we can take the values of x and y in the following way, which will satisfy the given equation :

x	5	5	5	5
y	–2	6	10	–4

In the graph paper plotting the points (5, –2), (5, 6), (5, 10) and (5, –4) and joining them by a scale we will get the following graph.

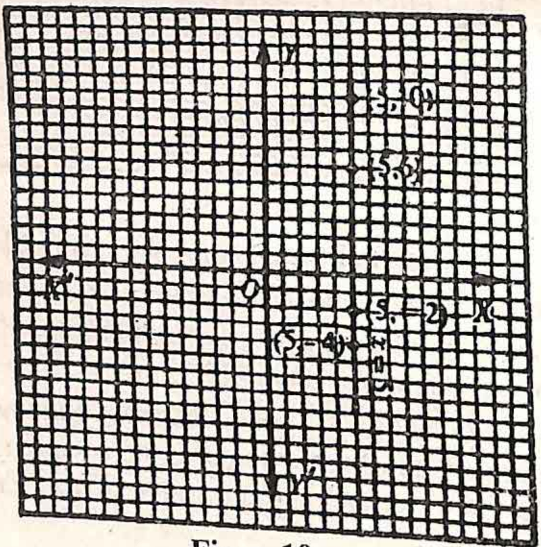


Fig - 10

(Graph of the equation $x = 5$)

Here obvserve the graph is parallel to the y-axis and is at a distance of 5 units from the origin in the positive direction of the x-axis. So to draw the graph of the equation x=5, it is sufficient to draw a straight line at a distance of 5 units from the origin in the positive direction of the x-axis and parallel to y-axix.

Similarly, the graph of the equation $x=10$ will be a straight line parallel to the y-axis and at a distance of 10 units from the origin in the positive direction of the x-axis.

The graph of $x=-10$ will be in the negative direction of the x-axis at a distance of 10 units from the origin and parallel to the y-axis.

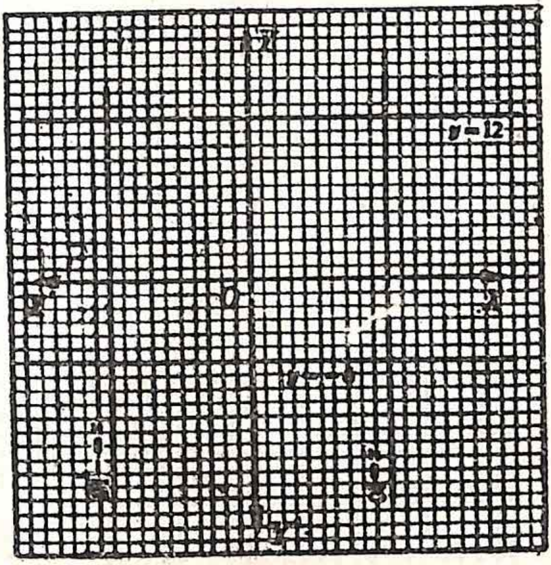


Fig - 11

Similarly, the graph of the equation $y = 12$ is a straight line parallel to the x-axis at a distance of 12 units. To obtain the graph of the equation $y = -6$, it is sufficient to draw a line parallel to the x-axis at a distance of 6 units from the origin in the negative direction of the y-axis.

From the above discussion it follows easily that for the equation $x = 0$, for any value of y the value of x is always equal to zero. So $x = 0$ will represent the equation of y-axis. The abscissa of any point on the y-axis is always zero.

Remember that the equation of x-axis is $y = 0$
and the equation of y-axis is $x = 0$.

Example 2.

Draw the graph of the equation $x - y = 10$

The given equation can be written as

$-y = -x + 10$

or $y = x - 10$

We are to put $x = 0$ or any integral value that gives integral value of y. Values of x and corresponding values of y are given in the following table :

x	10	0	5
y	0	−10	−5

Taking suitable origin on the graph.the two mutually perpendicular axes XOX' and YOY' are drawn.

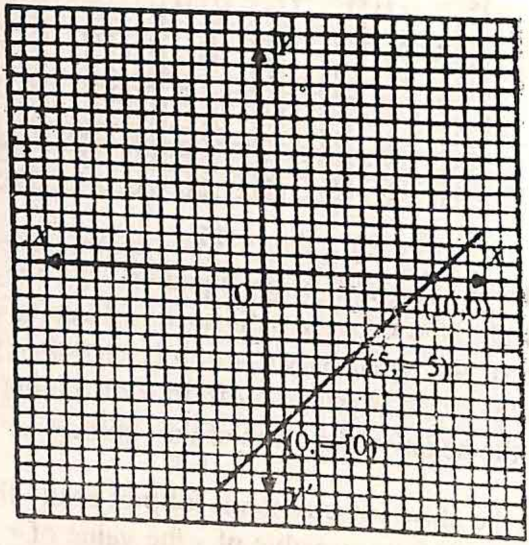


Fig - 12

(Graph of the equation $x - y = 10$)

Now taking the length of one side of the smallest square in the graph paper as unit, the points (10, 0) (0, −10) and (5, −5) are plotted.

If these points are joined with the help of a scale and the line segment is extended both ways ; it will be seen that all these points lie on a straight line.

This is the graph of the given equation. (Fig. No. 12)

Taking the axes according to convenicence and the length of sides of the two squares as uint draw the graph of the above equation.

Example 3.

Draw the graph of the equation $4x + 5y = 20$. Hence determine the length of the portion of the graph intercepted by the co-ordinate axes.

The given equation is $4x + 5y = 20$.

$$\text{or, } 5y = 20 - 4x$$

$$\text{or, } y = \frac{20 - x}{5}$$

Take the value of x as zero or an integer such that y will have integral values.

x	0	−5	−10	5
y	4	8	12	0

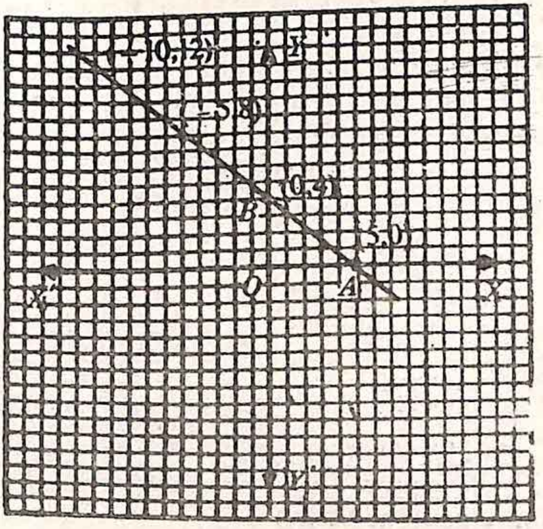


Fig - 13

Taking suitable origin, 0, two mutually perpendicular co-ordinate axes *XOX'* and *YOY'* are drawn. Taking the length of one side of the smallest square as unit the points (0, 4), (–5, 8), (–10, 12) and (5, 0) are plotted. Now with the help of a scale the points are joined together.

Graph of the equation $4x + 5y = 20$

Extending the line segment infinitely in both directions, the required graph will be obtained. (Fig. No. 13)

Here $OA = 5$ units

$OB = 4$ units

$$\therefore\ AB = \sqrt{OA^2 + OB^2}$$

$$= \sqrt{5^2 + 4^2}$$

$$= \sqrt{41}$$

Remark : Taking origin as centre and the intercept as radius draw an are of a circle to intersect x-axis (or y-axis). count the number of units from the origin upto the point where that are intersects *x*-axis (or *y*-axis). Thus you can determine the length of the intercept.

Example 4.

Taking the same axes, draw the graphs of equations $2x + 3y = 12$ and $2x - 3y = 0$ and hence find the co-ordinates of their point of intersection.

Solution. The two given equations are

$$2x + 3y = 12 \qquad \dots \quad (1)$$

$$\text{and } 2x - 3y = 0 \qquad \dots \quad (2)$$

From equation (1), we get

$$3y = 12 - 2x$$

$$\text{or, } y = \frac{12 - 2x}{3}$$

From this equation, we get

x	3	0	6
y	2	4	0

Again from equation (2), we have

$$-3y = -2x$$

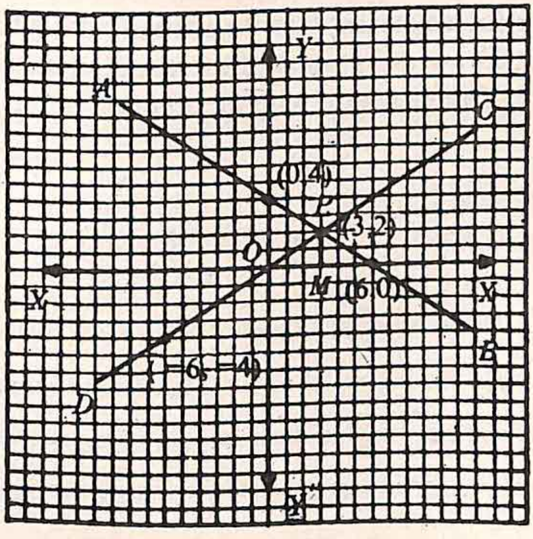
$$\text{or, } y = \frac{2x}{3}$$

From this equation, we get

x	0	3	–6
y	0	2	–4

Now XOX' and YOY', the two mutually perpendicular axes, are drawn. Taking the length of one side of the smallest square as unit the points (3, 2), (0, 4) and (6, 0) are plotted. Then these points are joined by a scale and the line segment thus obtained is extended along both the ends to get the graph *AB* of equation (1).

Again taking the same axes and unit the points (0, 0),



(3, 2) and (–6, –4) are plotted and the graph *CD* is drawn. the line *AB* and *CD* intersects at the point P.

From the point *P*, perpendicular *PM* is drawn of *x*-axis. On counting, it is found that

$$OM = 3 \text{ units and } MP = 2 \text{ units}$$

Thus the abscissa of the point of intersection is 3 and its ordinate is 2 and hence the co-ordinates are (3, 2).

Remark : The points of intersection of two graphs is the common point of the two graphs, i.e. the co-ordinates of this point will satisfy the equations of the two graphs. So the co-ordinates of the point of intersection of two graphs is the solution of the equations. So we find that by drawing the graphs of two equations and finding their point of intersection we can solve two equations.

Also we observe another fact. Two straight lines drawn on a plane will be of any one of the following types :

(i) They intersect each other, (ii) they are parallel to each other or (iii) they are coincident. Among these three cases only (i) gives a difinite solution of the two equations. You have learnt earlier how to solve linear simultaneous equations. Draw the graphs of the equations and compare their point of intersection with the solution of the equations.

Exercise 16

1. In which quardants or in which directions of which axes are the following points situated ?

(i) (3, 0), (ii) (0, 5), (iii) (–4, 0), (iv) (0, –6), (v) (5, 7), (vi) (–3, 4), (vii) (–5, –8), (viii) (7, –9)

2. Plot the above points a graph paper.

3. Plot the following points on a graph paper and join them by straight lines :

(i) (3, 4), (10, 12) ; (ii) (–5, 2), (3, 4) ; (iii) (–3, –4), (6, 8) ; (iv) (2, –4), (–6, 5) ; (v) (–1, –2), (–9, –10) ; (vi) (–5, –5), (10, 10).

4. Plotiting the three points (1, 2), (–1, 1) and (11, 7) on a graph verify that lie on the same line.

5. Draw the graphs of the following equations :

(i) $2x - 5y = 0$ (ii) $x = 3 - 4y$ (iii) $y = x - 5$

(iv) $\frac{x}{2} + \frac{y}{3} = 1$ (v) $5x + 3y = 8$ (vi) $x = 5$

(vii) $y = 4$ (viii) $\frac{x}{2} - \frac{y}{3} = 1$ (ix) $6x - 7y = 12$

(x) $x = -6$ (xi) $y = -3$ (xii) $7x - 3y = 21$

(xiii) $x - 7y = -7$ (xiv) $2y - 3x = 7$

6. Draw the graphs of the following equations and find their points of intersection :

(i) $x + y = 8$ and $x - y = 0$ (ii) $x = 4$ and $3x + 4y = 8$ (iii) $y = 5$ and $5x + 6y = 30$ (iv) $3x - 2y = 6$ and $2x + 3y = 4$ (v) $3x - 5y = 16$ and $2x - 9y = 5$

7. Draw the graph of the equation $\frac{x}{3} + \frac{y}{4} = 1$ and hence find the length of its portion intercepted between the co-ordinate axes.

8. Draw the graph of the equation $\frac{x}{7} - \frac{y}{8} = 1$ and find the area of the triangle formed by the graph and the coordinate axes.

CHAPTER 6

Ratio and Proportion

You have seen in arithmetic that the ratio of two quantities can be expressed in the form of a fraction. The numerator and the denominator of the fraction will respectively be the antecedent and the consequent of the ratio. The ratio of the quantities a and b is written as $a : b$. So observe that $a : b = \frac{a}{b}$.

Greater and smaller ratios :

If the antecedent is equal to the consequant, then the ratio is called an equality, otherwise it is an inequality e.g. $a : a$ is an equality, but $b : c$ is an inequality.

You know that ratio of two quantities = Antecedent : Consequent. If the value of the antecedent is greater than that of the consequent, then the ratio is called a ratio of greater inequality.

And if the antecedent is less than the consequent then the ratio is called a ratio of lesser inequality.

So if $a > b$, then $a : b$ is a ratio of greater inequality and $b : a$ is a ratio of lesser inequality.

If the terms of a ratio are multiplied or divided by a non-zero quantity, then its value remains unaltered.

for, $\mathrm{a : b} = \frac{\mathrm{a}}{\mathrm{b}} = \frac{\mathrm{ka}}{\mathrm{kb}} = \frac{\mathrm{a}}{\mathrm{k}} \cdot \frac{\mathrm{k}}{\mathrm{b}} = \frac{\frac{\mathrm{a}}{\mathrm{k}}}{\frac{\mathrm{b}}{\mathrm{k}}} = \frac{\mathrm{a}}{\mathrm{k}} : \frac{\mathrm{b}}{\mathrm{k}}$

Generally a ratio is expressed in lowes terms,

e.g. the ratio of 25 to 20 is $25 : 20 = \frac{25}{20} = \frac{5}{4} = 5 : 4$.

Inverse ratio : Between two ratios if the antecedent of one is equal to the consequent of the other and the antecedent of the later is equal to consequent of the former, then one is said to be the inverse ratio of the other.

For example, 5 : 3 and 3 : 5 are inverse to each other.

The inverse of $b : a$ is $a : b$.

If these two ratios are expressed in fraction, the two fractions will be $\frac{a}{b}$ and $\frac{b}{a}$. Observe that they are reciprocal to each other.

Compound or mixed ratio : The ratio whose antecedent is the product of the antecedents of two or more ratios and whose consequent is the product of the consequants of those ratios is called a compound or mixed ratio. For example the compound ratio of 2 : 3, 4 : 5 and 6 : 7 is $\frac{2 \times 4 \times 6}{3 \times 5 \times 7} = \frac{16}{35} = 16 : 35$

Similarly, the mixed ratio of $a : b$ and $c : d$ is $ac : bd$.

Example 1. If $x : y = 2 : 3$ then find the value of $4x - y : 2x + 3y$

Solution. $\frac{x}{y} = \frac{2}{3}$ $\quad \therefore \frac{x}{2} = \frac{y}{3} = p$ (say)

$\therefore x = 2p$ and $y = 3p$

Now $\frac{4x-y}{2x+3y} = \frac{4.2p-3p}{2.2p+3.3p} = \frac{5}{13} = 5 : 13$

Example 2. If $3x + 5y : 7x - 4y = 7 : 4$, what is the value of $x : y$?

Solution . $\frac{3x+5y}{7x-4y} = \frac{7}{4}$

or, $7(7x-4y) = 4(3x+5y)$ (by cross-multiplication)

or, $49x - 28y = 12x + 20y$

or, $49x - 12x = 20y + 28y$

or, $37x = 48y$ **or** $\frac{x}{y} = \frac{48}{37}$

$\therefore x : y = 48 : 37$

Example 3. Find the number which must be subtracted from both the terms of the ratio 2 : 3 to yield 3 : 5.

Solution. Let x be the number which should be subtracted from both the terms of 2 : 3. Then from the given condition.

$\frac{2-x}{3-x} = \frac{3}{5}$ or, $9 - 3x = 10 - 5x$

or, $-3x + 5x = 10 - 9$

or, $2x = 1$

$\therefore x = \frac{1}{2}$

$\therefore$ the required number is $\frac{1}{2}$.

Example 4. A certain ratio becomes 3 : 4 when 1 is added to both of its terms and it becomes 2 : 3 when 1 is subtracted from each of them. Find the ratio.

Solution. Let the required ratio be x : y.

So, according to the problem

$\frac{x+1}{y+1} = \frac{3}{4}$ **or,** $4x + 4 = 3y + 3$

or, $4x - 3y = 3 - 4$

or, $4x - 3y = -1$ .... (1)

Again, $\frac{x-1}{y-1} = \frac{2}{3}$ **or,** $3x - 3 = 2y - 2$

or, $3x - 2y = 3 - 2$

or, $3x - 2y = 1$ .... (2)

Now to find x and y we have to solve equations (1) and (2). Multiplying equations (1) and (2) by 2 and 3 respectively, we find,

$$\begin{array}{rl} & 8x - 6y = -2 \\ & 9x - 6y = 3 \\ \hline \text{(Subtracting)} & -x = -5 \end{array}$$

or, $x = 5$

In equation (1) substituting 5 for x, we get

$4.5 - 3y = -1$ **or,** $20 - 3y = -1$

or, $-3y = -21$ $\therefore y = 7$

$\therefore$ the required ratio = 5 : 7

Exercise 17

1. Find the compound ratio of the following :

(i). $3 : 4, 5 : 6, 7 : 8$ (ii). $2 : 3, 4 : 9, 5 : 11$

(iii) Rs. 2 : Rs. 3, 1cm 2mm : 14mm, 3 kg 500 gm : 3kg 400gm.

(iv) $a : b, c : d, e : f$ (v) $x^2 : yz, y^2 : zx$

(2) If $x : y = 3 : 4$ find $(3y - x) : (2x + y)$.

(3) If $a : b = 8 : 7$, find $(7 - 3b) : (11a - 9b)$.

(4) If $(5x - 3y) : (2x + 4y) = 11 : 12$, find $x : y$.

(5) If $(3a + 7b) : (5a - 3b) = 5 : 3$, find the value of $a{:}b$.

(6) $(3x - 2y) : (x + 3y) = 5 : 6$, find the value of $\frac{2x-5y}{3x+4y}$.

(7) If $(7x - 5y) : (3x + 4y) = 7 : 11$, find the value of $\frac{5x-3y}{6x+5y}$.

(8) What must be added to either term of $2 : 5$ to get $6 : 11$?

(9) What must be subtracted from either term of $4 : 7$ get $2 : 5$?

(10) What must be added to either term of $x{:}y$ to get $a{:}b$?

(11) What must be subtracted from either term of $p{:}q$ to get $m : n$?

Proportion :

If two ratios are equal, then the four quantities composing then are said to be proportional m. for example, Rs. 4 : Rs. 6 = 2 : 3 ; 8 gm : 12 gm = 2 : 3.

∴ the ratio of Rs. 4 to Rs. 6 is equal to the ratio of 8 gm to 12 gm.

These are called **proportion.**

Of four terms, if the ratio of first and second is equal to ratio of third and fourth, then the four terms are called proportional, i.e., if $a : b = c : d$, then a, b, c and d are called proportional.

This is also denoted by $a : b :: c : d$

Here a and d are called *extremes* and b and c are called *means*, i.e., the first and the fourth terms are called extremes and the second and the third terms are called means. The term d is called the *fourth proportional*. b is called mean proportional of a and c.

Continued Proportion : Three quantities a, b and c are said to be a continued proportion, if $a : b :: b : c$, i.e., if $\frac{a}{b} = \frac{b}{c}$ and we find the if a, b, c, are in continued proportion, then $ac = b^2$, so if three numbers are in continued proportion, then the product of the first and third is equal to the square of the mean.

For example, 2,4 and 8 are in continued proportion. More than three numbers may be in continued proportion. If $a : b = b : c = c : d$, then a, b, c and d are in continued proportion, e.g, 2, 4, 8, 16, 32 are in continued proportion.

Some useful properties of proportion :

(i) Alternendo : If $a : b = c : d$ then $a : c = b : d$

Proof : $\frac{a}{b} = \frac{c}{d}$

Multiplying both sides by $\frac{b}{c}$, we have

$$\frac{a}{b}\times\frac{b}{c} = \frac{c}{d}\times\frac{b}{c}$$

$$\text{or, } \frac{a}{c}=\frac{b}{d} \text{ or, } a:c=b:d$$

(ii) Invertendo : If $a:b=c:d$, then $b:a=d:c$,

i.e., if two ratios are equal, their inverses are also equal.

Proof : Since $a:b=c:d$ $\therefore \frac{a}{b}=\frac{c}{d}$

or, $1\div\frac{a}{b}=1\div\frac{c}{d}$ i.e., $\frac{b}{a}=\frac{d}{c}$

$$\therefore b:a=d:c$$

(iii) **Componendo :** If $a:b=c:d$, then $(a+b):b=(c+d):d$

Proof : $\frac{a}{b}=\frac{c}{d}$ $\therefore$ Adding 1 to both sides, we get

$\frac{a}{b}+1=\frac{c}{d}+1$

or, $\frac{a+b}{b}=\frac{c+d}{d}$ i.d., $(a+b):b=(c+d):d$

(iv) Dividendo : If $a:b=c:d$. then $(a-b):b=(c-d):d$

Proof : $\frac{a}{b}=\frac{c}{d}$ $\therefore \frac{a}{b}-1=\frac{c}{d}-1$ (subtracting 1 from both sides)

or, $\frac{a-b}{b}=\frac{c-d}{d}$ i.e., $(a-b)=(c-d):d$.

(v) **Componendo and Dividendo :**

If $a:b=c:d$ then $(a+b):(a-b)=(c+d):(c-d)$

Proof : Since $a:b=c:d$

$\therefore \frac{a+b}{b}=\frac{c+d}{d}$ .... (1)

and $\frac{a-b}{b}=\frac{c-d}{d}$ .... (2)

[from (iii) and (iv)],

Dividing each side of (1) by the corresponding side of (2) we have

$\frac{a+b}{a-b}=\frac{c+d}{c-d}$ i.e., $(a+b+ : (a-b) = (c+d) : (c-d)$

Alternative Proof : Let $\frac{a}{b}=\frac{c}{d}=k$. $\therefore$ $a = kb$ and $c = kd$.

$$\therefore \frac{a+b}{a-b}=\frac{kb+b}{kb-b}=\frac{k+1}{k-1} \text{ and } \frac{c+d}{c-d}=\frac{kd+d}{kd-d}=\frac{k+1}{k-1}$$

$$\therefore \frac{a+b}{a-b}=\frac{c+d}{c-d}$$

(vi) **Addendo :** If $a : b = c : d = e : f$, then each ratio is equal to $(a + c + e) : (b + d + f)$

i.e., $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{a+c+e}{b+d+f}$ and

in general $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=$ .... $=\frac{a+c+e......}{b+d+f......}$

Proof : Since $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k$

$$\therefore a = bk, c = dk, e = fk$$

$$\therefore \frac{a+c+e}{b+d+f}=\frac{bk+dk+fk}{b+d+f}$$

$$=\frac{k(b+d+f)}{b+d+f}=k$$

$$\therefore \frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{a+c+e}{b+d+f}$$

[*Remark* : $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}\dots=\frac{la+mc+ne+....}{lb+md+nf+....}$, when values c all the quantities l, m, n...... are not zero.]

Example 1. Find the fourth proportional of 6, 9 and 12.

Solution. Let x be the fourth proportional. Then 6, 9, 12 and x are proportional.

$$\therefore \frac{6}{9}=\frac{12}{x} \text{ or, } 6x = 108$$

$$\therefore x = 18$$

Example 2. find the third proportional of $2a^2$, $3ab$.

Solution. Let x be the third proportional. Then $2a^2$, $3ab$ and x are in continued proportion.

$$\therefore \frac{2a^2}{3ab} = \frac{3ab}{x} \text{ or, } 2a^2x = 3ab \times 3ab$$

$$\therefore x = \frac{3ab \times 3ab}{2a^2}$$

$$\therefore x = \frac{9b^2}{2}$$

Hence the required third proportional is $\frac{9b^2}{2}$.

Example 3. Find the mean proportion between xy^2 and xz^2.

Solution. Let m be the mean proportional between xy^2 and xz^2

then $\frac{xy^2}{m} = \frac{m}{xz^2}$ or $m^2 = x^2y^2z^2$

$$\therefore m = \pm\sqrt{x^2y^2z^2} = \pm xyz.$$

$\therefore$ the required mean proportional is $= \pm xyz$.

Example 4. What should be added to each of 2, 4, 6 and 10 so that the sums will be proportional ?

Solution. Let x be the required number.

Then $2 + x.4 + x, 6 + x$ and $10 + x$ will be proportional,

i.e., $\frac{2+x}{4+x} = \frac{6+x}{10+x}$

or, $(2+x)\ (10+x) = (6+x)\ (4+x)$

or, $20 + 12x + x^2 = 24 + 10x + x^2$

or, $12x - 10x = 24 - 20$

or, $2x = 4$

$\therefore x = 2$

Hence the required number is 2.

Exercise 18

1. Find the fourth proportional of the following :

(i) 3, 4, 6. (ii) 3, 5, 9. (iii) a, b, c. (iv) ab, bc, cd. (v) x^2y, y^2z, xz^2.

2. Find the third proportional of the following :

(i) 8, 12. (ii) a, b. (iii) x^3y^2, y^2x

3. Find the mean proportional of the following :

(i) –4, –16. (ii) $\frac{1}{3}$, $\frac{1}{27}$. (iii) a^2, b^2.

(iv) x^3y, xy. (v) $(x - y)^2$, $(x + y)^2$.

4. What number must be added to each of 6, 7, 15, 17 so that the sums are proportional ?

5. The first term of 5 numbers in continued proportion is 2 and its second term is 6, find the fifth.

6. What must be subtracted from each of 13, 17, 23 and 31 so that the results are proportional ?

7. What must be added to each of a, b, c and d so that the sums are proportioinal ?

Application of the properties of proportion :

Example 1. If $a : b = c ; d$, prove that $(a^2+b^2)(b^2+d^2) = (ab+cd)^2$

Solution. Let $\frac{a}{b} = \frac{c}{d} = k \therefore a = bk$ and $c = dk$

$$\text{L.H.S.} = (a^2 + c^2)(b^2 + d^2) = (b^2k^2 + d^2k^2)(b^2 + d^2)$$

$$= k^2(b^2 + d^2)^2 = \{k(b^2 + d^2)\}^2$$

$$\text{R.H.S.} = (ab + cd)^2 = (bk.b + dk.d)^2$$

$$= k^2(b^2 + d^2)^2 = \{k(b^2 + d^2)\}^2$$

$$\therefore \text{ L.H.S.} = \text{R.H.S.}$$

$$\therefore (a^2 + c^2)(b^2 + d^2) = (ab + cd)^2$$

Example 2. If $a : b = c : d$, prove that

$$\frac{a^2 + b^2}{a^2 - b^2} = \frac{c^2 + d^2}{c^2 - d^2}$$

Solution. Since $\frac{a}{b} = \frac{c}{d} \quad \therefore \frac{a^2}{b^2} = \frac{c^2}{d^2}$ (squaring both sides)

By componendo and dividendo we have,

$$\frac{a^2 + b^2}{a^2 - b^2} = \frac{c^2 + d^2}{c^2 - d^2}$$

Alternative proof : Let $\frac{a}{b} = \frac{c}{d} = k$ (say) $\therefore a = bk, c = dk$

$$\text{L.H.S.} = \frac{a^2 + b^2}{a^2 - b^2} = \frac{b^2k^2 + b^2}{b^2k^2 - b^2} = \frac{b^2(k^2 + 1)}{b^2(k^2 - 1)} = \frac{k^2 + 1}{k^2 - 1}$$

$$\text{R.H.S.} = \frac{c^2 + d^2}{c^2 - d^2} = \frac{d^2k^2 + d^2}{d^2k^2 - d^2} = \frac{d^2(k^2 + 1)}{d^2(k^2 - 1)} = \frac{k^2 + 1}{k^2 - 1}$$

$$\therefore \frac{a^2 + b^2}{a^2 - b^2} = \frac{c^2 + d^2}{c^2 - d^2}$$

Example 3. If $a : b = b : c = c : d$, then prove that

$$(a + b) : (c + d) = (a^2 + b^2 + c^2) : (b^2 + c^2 + d^2)$$

Solution. Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = k$ (say)

$$\therefore c = dk, b = ck, = dk.\,k = dk^2$$

$$a = bk = dk^2.\,k = dk^3$$

$$\text{L.H.S.} = \frac{a+b}{c+d} = \frac{dk^3 + dk^2}{dk+d} = \frac{k^2(dk+d)}{dk+d} = k^2$$

$$\text{R.H.S.}\ \frac{a^2+b^2+c^2}{b^2+c^2+d^2} = \frac{d^2k^6 = d^2k^4 + d^2k^2}{d^2k^4 + d^2k^2 + d^2}$$

$$= \frac{k^2(d^2k^4 + d^2k^2 + d^2)}{(d^2k^4 + d2k^2 + d^2)} = k^2$$

$$= \text{L.H.S}$$

Example 4. If a, b, c, d are in continued proportion, then prove that $(b-c)^2 + (c-a)^2 + (b-d)^2 = (a-d)^2$

Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = k$ (say)

$c = dk$

$b = ck = dk.k = dk^2$

$a = bk = dk.d^2 = dk^3$

$$\text{L.H.S.} = (b-c)^2 + (c-a)^2 + (b-d)^2$$
$$= (dk^2 - dk)^2 + (dk - dk^3)^2 + (dk^2 \neq d)^2$$
$$= \{dk(k-1)\}^2 + \{-dk(k^2-1)\}^2 + \{d(k^2-1)\}^2$$
$$= d^2k^2(k-1)^2 + d^2k^2(k^2-1)^2 + d^2(k^2-1)^2$$
$$= d^2\{k2(k^2-2k+1) + k^2(k^4-2k^2+1) + (k^4-2k^2+1)\}$$
$$= d^2k^4 - 2k^3 + k^2 + k^6 - 2k^4 + k^2 + k^4 - 2k^2 + k$$
$$= d^2k^6 - 2k^3 + 1 = d^2(k^6 - 2k^3 + 1) = d^2(k^3-1)^2$$
$$= (dk^3 - d)^2 = (a-d)^2$$

Example 5. Prove, by using proportion, that

$$\frac{x+2a}{x-2a} + \frac{x+2b}{x-2b} = 2 \text{ when } x = \frac{4ab}{a+b}$$

Solution.

It is given that

$$x = \frac{4ab}{a+b} \qquad \dots \qquad (1)$$

Dividing both sides by $2a$, we have

$$\frac{x}{2a} = \frac{2b}{a+b}$$

Now $x : 2a : 2b : : 2b : (a + b)$

$\therefore (x + 2a) : (x - 2a) : : (3b + a) : (b - c)$

Again, $x : 2b : : 2a : (a + b)$

$\therefore (x + 2b) : (x - 2b) : : (3a + b) : (a - b)$

$$\therefore \frac{x + 2a}{x - 2a} + \frac{x + 2b}{x - 2b} = \frac{3b + a}{b - a} + \frac{3a + b}{a - b}$$

$$= \frac{3b + a - 3b - b}{b - a}$$

$$= \frac{2b - 2a}{b - a}$$

$$= 2.$$

Exercise 19

1. If $a : b = c : d$, prove the following :
    - (i) $a^2 + b^2 : a^2 - b^2 = ac + bc : ac - bd$
    - (ii) $a^2 + ab + b^2 : a^2 - ab + b^2 = c^2 + cd + d^2 : c^2 - cd + d^2$
    - (iii) $\sqrt{a^2 + c^2} : \sqrt{b^2 + d^2} = pa + qc : pb + qd$
    - (iv) $a^2 + c^2 : b^2 + d^2 = c(a + c) : d(b + d)$
2. If $\frac{m}{a} = \frac{n}{b}$, prove that $(m^2 + n^2)(a^2 + b^2) = (am + bn)^2$
3. If $\frac{a}{b} = \frac{x}{y}$, prove that $(a + b)(a^2 + b^2)x^3 = (x + y)(x^2 + y^2)a^3$
4. If $x : a = y : b = x : c$, prove that
    - (i) $\frac{x^3}{z^2} + \frac{y^3}{b^2} + \frac{z^3}{c^2} = \frac{(x + y + z)^3}{(a + b + c)^2}$
    - (ii) $\frac{x^3 + y^3 + z^3}{a^3 + b^3 + c^3} = \frac{xyz}{abc}$
    - (iii) $(a^2 + b^2 + c^2)(x^2 + y^2 + z^2) = (ax + by + cz)^2$
5. If $\frac{ay - bx}{c} = \frac{cx - az}{b} = \frac{bz - cy}{a}$, prove that $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$
6. If $a : b = b : c$, prove the following :
    - (i) $\frac{a}{c} = \frac{a^2 + b^2}{b^2 + c^2}$
    - (ii) $\left(\frac{a + b}{b + c}\right)^2 = \frac{a^2 + b^2}{b^2 + c^2}$

(iii) $a^3b^3c^3\left(\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}\right)=a^3+b^3+c^3$

(iv) $(a+b+c)(a-b+c)=a^2+b^2+c^2$

(v) $\frac{abc(a+b+c)^3}{(ab+bc+ca)^3}=1$

7. If a, b, c and d be in continued proportion, prove the following :

(i) $a : d = a^3+b^3+c^3 : b^3+c^3+d^3$

(ii) $(a^2-b^2)(c^2-d^2)=(b^2-c^2)^2$

(iii) $(a+c)^2 : (b+d)^2 = (a^2+b^2+c^2) : (b^2+c^2+d^2)$

(iv) $(a^2+b^2+c^2)(b^2+c^2+d^2)=(ab+bc+cd)^2$

8. If $a : b = c : d = e : f$, prove that

(i) each ratio is equal to $\frac{5a-7c-13e}{5b-7d-13f}$

(ii) $(a^2+b^2)(ce+df)^2$

$=(e^2+f^2)\ (ac+bd)^2$

(iii) $\frac{ac+ce+ae}{bd+df+bf}=\frac{a^2}{b^2}$

(iv) $\frac{a^2+c^2+e^2}{b^2+d^2+f^2}+\frac{ce}{df}$

(v) $(a^2+c^2+e^2)(b^2+d^2+f^2)=(ab+cd+ef)^2$

(iv) $\frac{2a^2+3c^2+5e^2}{2b^2+3d^2+5f^2}=\frac{ac}{bd}$

Miscellaneous Examples on Proportion :

Example 1. If $\frac{a}{x+y-z} = \frac{b}{y+z-x} = \frac{c}{z+x-y}$, show that each ratio is equal to $\frac{a+b+c}{x+y+z}$

Solution. $\frac{a}{x+y-z} = \frac{b}{y+z-x} = \frac{c}{y+x-z}$

$\therefore$ by addendo, we have

$$\frac{a}{x+y-z} = \frac{b}{y+z-x} = \frac{c}{y+x-z} =$$

$$\frac{a+b+c}{x+y-z+y+z-x+z+x-y} = \frac{a+b+c}{x+y+z}$$

Example 2. If $\frac{x^2-yz}{a} = \frac{y^2-zx}{b} = \frac{z^2-xy}{c}$, prove that

$$(a+b+c)(x+y+z) = ax+by+cz$$

Solution. Since $\frac{x^2-yz}{a} = \frac{y^2-zx}{b} = \frac{z^2-xy}{c}$

$\therefore$ by invertendo, we have

$$\frac{a}{x^2-yz} = \frac{b}{y^2-zx} = \frac{c}{x^2-xy} = k \text{ (say)}$$

$\therefore \quad a = k(x^2-yz),\ b = k(y^2-zx),\ c = k(z^2-xy)$

L.H.S. $= (a+b+c)(x+y+z)$

$= [k(x^2-yz) + k(y^2-zx) + k(z^2-xy)](x+y+z)$

$= k(x^2+y^2+z^2-xy-yz-zx)(x+y+z)$

$= k(x^3+y^3+z^2-3xyz)$

R.H.S. $= (ax+by+cz)$

$= k(x^2-yz)x + k(y^2-zx)y + k(z^2-xy)z$

$= k(x^3+y^3+z^3-3xyz)$ = L.H.S. (Proved)

Example 3. If $\frac{x}{lm-n^2} = \frac{y}{mn-l^2} = \frac{z}{nl-m^2}$, prove that

$$lx+my+nz=0$$

Solution. Let $\frac{x}{lm - n^2} = \frac{y}{mn - l^2} = \frac{z}{nl - m^2}, = k$ (say)

$\therefore x = k(lm - n^2), y = k(mn - l^2), z = k(nl - m^2)$

L.H.S. $= lx + my + nz$

$= lk(lm - n^2) + mk(mn - l^2) + nk(nl - m^2)$

$= k(l^2m - ln^2 + m^2n - ml^2 + n^2l - nm^2)$

$=$ k.0 = 0 = R.H.S. (Proved)

Example 4.

If $\frac{a + b - c}{a + b} = \frac{b + c - a}{b + c} = \frac{c + a - b}{c + a}$ and $a + b + c \neq 0$,
Prove that $a = b = c$.

Solution. $\frac{a + b - c}{a + b} = \frac{b + c - a}{b + c} = \frac{c + a - b}{c + a}$

or, $1 - \frac{c}{a + b} = 1 - \frac{a}{b + c} = 1 - \frac{b}{c + a}$

or, $- \frac{c}{a + b} = - \frac{a}{b + c} = - \frac{b}{c + a}$

or, $\frac{c}{a + b} = \frac{a}{b + c} = \frac{b}{c + a}$

or, $\frac{a + b}{c} = \frac{b + c}{a} = \frac{c + a}{b}$

or, $\frac{a + b}{c} + 1 = \frac{b + c}{a} + 1 = \frac{c + a}{b}$ [Adding 1 with each ratio]

or, $\frac{a + b + c}{c} = \frac{b + c + a}{a} = \frac{c + a + b}{b}$

or, $\frac{1}{c} = \frac{1}{a} = \frac{1}{b}$ [$\because a + b + c \neq 0, \therefore$ dividing each ratio by $a + b + c] = \frac{2x}{b^2 + c^2 - a^2}$

$\therefore a = b = c$

Example 5.

If $\frac{bz + cy}{a} = \frac{cx + az}{b} = \frac{ay + bx}{c}$, prove that

$$\frac{x}{a(b^2 + c^2 - a^2)} = \frac{y}{b(c^2 + a^2 - b^2)} = \frac{x}{c(a^2 + b^2 - c^2)}$$

Solution : $\dfrac{bx + cy}{a} = \dfrac{cx + az}{b} = \dfrac{ay + bx}{c}$

$$\frac{bc\left(\frac{z}{c} + \frac{y}{b}\right)}{a} = \frac{ca\left(\frac{x}{a} + \frac{z}{c}\right)}{b} = \frac{\left(\frac{y}{b} + \frac{x}{a}\right)}{b}$$

Dividing each ratio by abc, we have

$$\therefore \frac{\frac{z}{\cancel{x}} + \frac{y}{b}}{a^2} = \frac{\frac{x}{a} + \frac{z}{c}}{b^2} = \frac{\frac{y}{b} + \frac{z}{a}}{c^2}$$

$$\therefore \frac{\frac{z}{\cancel{x}} + \frac{y}{b}}{a^2} = \frac{\frac{x}{a} + \frac{z}{c}}{b^2} = \frac{\frac{y}{b} + \frac{z}{a}}{c^2}$$

$$= \frac{1.\left(\frac{x}{a} + \frac{z}{c}\right) + 1.\left(\frac{y}{b} + \frac{x}{a}\right) - 1.\left(\frac{z}{c} + \frac{y}{b}\right)}{1.b^2 + 1.c^2 - 1.a^2}$$

$$= \frac{1.\left(\frac{y}{b} + \frac{x}{a}\right) + 1.\left(\frac{z}{c} + \frac{y}{b}\right) - 1.\left(\frac{x}{a} + \frac{z}{c}\right)}{1.c^2 + 1.a^2 - 1.b^2}$$

$$= \frac{1.\left(\frac{z}{c} + \frac{y}{b}\right) + 1.\left(\frac{x}{a} + \frac{z}{c}\right) - 1.\left(\frac{y}{b} + \frac{x}{a}\right)}{1a^2 + 1.b^2 - 1.c^2}$$

$$= \frac{2y}{c^2 + a^2 - b^2} = \frac{2z}{a^2 + b^2 - c^2}$$

$$\therefore \frac{x}{a(b^2+c^2-a^2)} = \frac{y}{b(c^2+a^2-b^2)} = \frac{x}{c(a^2+b^2-c^2)}$$

Alternative Method :

$$\frac{bz+cy}{a} = \frac{cx+az}{b} = \frac{ay+bx}{c}$$

or, $\dfrac{abz+acy}{a^2} = \dfrac{bcx+abz}{b^2} = \dfrac{acy+bcx}{c^2}$

or, $\dfrac{2bcx}{b^2+c^2-a^2} = \dfrac{2acy}{c^2+a^2-b^2} = \dfrac{2abz}{a^2+b^2-c^2}$

or, $\dfrac{2bcx}{a(b^2+c^2-a^2)} = \dfrac{2acy}{b(c^2+a^2-b^2)} = \dfrac{2abz}{c(a^2+b^2-c^2)}$

or, $\dfrac{x}{a(b^2+c^2-a^2)} = \dfrac{y}{b(c^2+a^2-b^2)} = \dfrac{z}{c(a^2+b^2-c^2)}$

Exercise 20

1. If $\frac{x}{a+b-c} = \frac{y}{b+c-a} = \frac{z}{c+a-b}$, prove that each ratio is equal to $\frac{x+y+z}{a+b+c}$

2. If $\frac{x}{y+z} = \frac{y}{z+x} = \frac{z}{x+y}$ and $x+y+z \neq 0$, prove that $x = y = z$.

3. If $(a+b+c)p = (b+c-a)q = (c+a-b)r = (a+b-c)s$, prove that $\frac{1}{q} + \frac{1}{r} + \frac{1}{s} = \frac{1}{p}$

4. If $\frac{a+b}{b+c} = \frac{c+d}{d+a}$, prove that either $c = a$ or $a+b+c+d = 0$.

5. If $\frac{x}{(b-c)(b+c-2a)} = \frac{y}{(c-a)(c+a-2b)} = \frac{z}{(a-b)(a+b-2c)}$ prove that $x + y + z = 0$.

6. If $\frac{b}{a+b} = \frac{a+c-b}{b+c-a} = \frac{a+b+c}{2a+b+2c}$, prove that $\frac{a}{2} = \frac{b}{3} = \frac{c}{4}$.

7. If $\frac{x}{b+c-a} = \frac{y}{a+a-b} = \frac{z}{a+b-c}$, find the value of $(b-c)x + (c-a)y + (a-b)z$.

8. If $\frac{bz-cy}{b-c} = \frac{cx-az}{c-a}$, show that each ratio is equal to $\frac{ay-bx}{a-b}$

9. If $\frac{x}{b+c} = \frac{y}{c+a} = \frac{z}{a+b}$, show that $\frac{a}{y+z-x} = \frac{b}{z+x-y} = \frac{c}{x+y-z}$

10. If $\frac{x}{y} = \frac{a+2}{a-2}$, find the value of $\frac{x^2-y^2}{x^2+y^2}$

11. If $\frac{x}{y+z} = \frac{y}{z+x} = \frac{z}{x+y}$, prove that the value of each of the ratio is either $\frac{1}{2}$ or -1.

12. If $\frac{x}{ax+by+cz} = \frac{y}{ay+bx+cx} = \frac{x}{az+bx+cy}$ and $x+y+z \neq 0$, show that the value of each ratio is $\frac{1}{a+b+c}$

Miscellaneous Exercise 1

Resolve into factors :

1. (i) $x^4 + 4y^4$

[Hint : Express it as sum of two squares and use the formula, $a^2 + b^2 = (a + b)^2 - 2ab$ and finally, $a^2 - b^2 = (a + b)(a - b)$.]

(ii) $4x^4 + 81$ (iii) $a^8 - 16$ (iv) $x^4 + x^2y^2 + y^4$.

[Hint : Rearrange it in powers of x or y if necessary, and use the formula $a^2 + b^2 = (a + b)^2 - 2ab$ or, $a^2 + b^2 = (a - b)^2 + 2ab$ and finally use $a^2 - b^2 = (a + b)(a - b)$].

(v) $x^4 - 7x^2 + 1$ (vi) $p^4 - 11p^2q^2 + q^4$ (vii) $m^8 + m^4 + 1$

(viii) $a^2 + b^2 - c^2 - 2ab$ (ix) $b^2 - c^2 - a^2 + 2ac$

(x) $3a(3a + 2c) - 4b(b + c)$ (xi) $x(x - 1) - y(y - 1)$

(xii) $a^2 - 6ab + 12bc - 4c^2$ (xiii) $3a^2 + 4ab + b^2 - 2ac - c^2$

(xiv) $a^2 - 3b^2 - c^2 - 2ab + 4bc$

(xv) $a^2 - 9b^2 + 4c^2 - 25d^2 - 4ac + 30bd$

2. (i) $x^6 + 27a^6$ (ii) $8a^4 - 125ab^3$ (iii) $8(x - 3)^3 + 343$

(iv) $m^3 - 3m^2n + 3mn^2 - 9n^3$ (v) $m^6 - n^6$

[Hint : You may use formula for $a^3 - b^3$ or $a^2 - b^2$ but it is convenient to use $a^2 - b^2 = (a + b)(a - b)$ first.]

(vi) $1 - x^{12}$ (vii) $a^3 - 6a + 4$

[Hint : Resolving 4 as (12 – 8) write the expression in form of a3 – b3 and the resolve into factors.]

(viii) $a^3 - 19a - 30$ (ix) $a^3 - a^2 - 18$ (x) $a^3 - 2ax^2 + 1$.

3. (i) $a^2 + 5a + 6$ (ii) $a^2 + 5a - 6$ (iii) $2a^3 + 24a^2 + 54a$

(iv) $a^2 + 2a - (a - 3)(a - 1)$ (v) $(a - b)^2 - a + b - 6$

(vi) $a^2 - 7ab - 18b^2 + a + 2b$

(vii) $(x^2 - 2x)^2 - 11(x^2 - 2x) + 24$

(viii) $(x^2 + 1)^2 - (x^2 - 1) - 4x^2$

[Hint : Write $(x^2 + 1)^2$ as $(x^2 - 1)^2 + 4x^2$]

(ix) $(x^2 - 1)^2 + 8x(x^2 + 1) + 19x^2$

(x) $x(x + 1)(x - 2)(x - 3) - 7$.

Miscellaneous Exercise 2

1. Find the H.C.F. (by factorisation) :
   (i) $x^2 - 4, 8 - x^2, 6 - x - x^2$

   [Observe that $2 - x$ can be wirtten as $-(x - 2)$; so, $2 - x$ and $x - 2$ have a common factor.]

   (ii) $5x - 4 - x^2, 2 - 3x + x^2, x^2 + 2x - 3$
   (iii) $a^2+b^2-c^2-2ab, a^2-b^2+c^2+2ac, b^2+c^2-a^2-2bc$
   (iv) $a^2-b^2-c^2+2bc, b^2-c^2-a^2+2ac, c^2-a^2-b^2-2ab$
   (v) $a^2+ab+bc+ca, b^2+ab+bc+ca, a^2-b^2$
   (vi) $x^4-8xy^3, x^4y-4x^2y^3, x^3y + 8y^4, x^4+4x^2y^2+16y^4$
   (vii) $x^3 - 8, x^2 - 5x + 6, x^3 - 4x^2 + 4x$
   (viii) $a^3b^2 - a2b^3, a^3b^2 - ab^4, a^4b - ab^4$
   (ix) $2a^2 - 4a, 3a^4 - 12a^2, 2a^5 - 2a^4 - 4a^3$
   (x) $2a^2x - a^3, 4ax^2 - a$
   (xi) $x^4 + 6x^2 + 5, x^3 - 3x^2 + x - 3.$
2. Find the H.C.F. (by division) :
   (i) $x^2 + 9x^2 + 23x - 12, x^3 - 10x^2 + 28x - 15$
   (ii) $3x^3 - 7x^2 - 18x - 8, 2x^3 - 3x^2 - 17x - 12$
   (iii) $x^3 + 6x^2 + 11x + 6, x^3 + 6x^2 + 8x$
   (iv) $6x^4 + 10x^3 + 10x^2 + 4x, 4x^5 + 10x^4 + 10x^3 + 6x^2$
   (v) $3x^5 - 10x^3 + 15x - 8, x^5 + 2x^4 - 6x^3 - 4x^2 + 13x - 6$
   (vi) $4x^3 - 12x - 8, 6x^3 - 24x^2 + 30x - 12$
   (vii) $x^3 + 4x^2 + 4x + 3, x^3 + 2x^2 - 2x + 3$
   (viii) $x^3 - x^2 - x - 15, x^3 - 3x^2 - 3x + 9$
   (ix) $2x^6 - 2x^5 - 6x^4 + 4x^3, 6x^6 + 6x^5 - 12x^2 + 6x$
   (x) $x^4 - 10x^3 + 35x^2 - 50x + 24, x^3 + x^2 + 5x - 7.$
3. Find the L.C.M. :
   (i) $x^2 - 3x + 2, x^2 - 5x + 6, 4x - 3 - x^2$
   (ii) $x^2 + 7x + 10, 6x + x^2 - x^3, x^4 - 15x^2 + 2x^3$
   (iii) $6x^2 - x - 1, 3x^2 + 7x + 2, 2x^2 + 3x - 2$
   (iv) $x(4 - x^2), x^4 + 6x^3 + 8x^2, x^2 + 2x - 8$
   (v) $a^2 + ab + bc + ca, b^2 + bc + ca + ab, c^2 + ab + bc + ca$
   (vi) $3x^2 + 6x + 3, 2x^2 + 6x + 4, 6ax + 12a$
   (vii) $2x^2 - 8, 3x - 6, x^2 - 4x + 4$
   (viii) $a^2-b^2-c^2 + 2bc, b^2 - c^2 - a^2 + 2ac, c^2 - a^2 - b^2 + 2ab$
   (ix) $x^3 - 1, x^3 + 1, x^4 + x^2 + 1, x^2 - 1$
   (x) $x^2 - ax, a^2 - ax, x^2a - a^2x, x^2 - a^2.$

Miscellaneous Exercise 3

1. Solve the following by the method of comparison (expressing x in terms of y) :

(i) $5x + 2y = 7$
$3x - 2y = 2$

(ii) $\frac{x}{3} + \frac{y}{4} = 1$
$2x + 4y = 11$

(iii) $7x - 3y = 10$
$x - 2y + 19 = 0$

(iv) $x + \frac{2}{y} = 7$
$2x - \frac{6}{y} = 9$

(v) $\frac{1}{x} + \frac{1}{y} = 8$
$\frac{1}{x} - \frac{1}{y} = -2$

(vi) $\frac{4}{x} - \frac{y}{2} = -1$
$\frac{8}{x} + 2y = 10$

(vii) $ax + by = c$
$px + qy = r.$

2. Solve the following by the method of comparison (expressing y in terms of x) :

(i) $3x - 2y = 4$
$x + 2y = 4$

(ii) $4x + 5y = -1$
$x - 3y = 4$

(iii) $5x - 4y - 23 = 0$
$2x + 3y = 0$

(iv) $3x + 4y - 25 = 0 = 4x - 3y$

(v) $\frac{x+1}{y+1} = \frac{3}{4}$
$\frac{x-1}{y-1} = \frac{2}{3}$

(vi) $\frac{x+y}{xy} = \frac{9}{20}$
$\frac{x-y}{xy} = -\frac{1}{20}$

(vii) $2{\cdot}1x + 4{\cdot}5y = 13{\cdot}5$
$3{\cdot}7x - 0{\cdot}1y = -0{\cdot}3.$

3. Solve the following by the method of substitution :

(i) $x + y = a$
$x - y = b$

(ii) $2x + y = 8$
$3x - 2y = 5$

(iii) $\frac{3}{x} + \frac{2}{y} = 13$
$\frac{7}{x} + \frac{3}{y} = 27$

(iv) $\frac{x+y}{xy} = 3$
$\frac{x-y}{xy} = 1$

(v) $2x - 3y + 10 = 0$
$\frac{x}{2} + \frac{y}{3} - 4 = 0$

(vi) $\frac{9}{x} - \frac{4}{y} = 2$
$\frac{9}{x} + \frac{4}{y} = 5$

(vii) $\frac{xy}{x + y} = 2$
$\frac{xy}{x - y} = 1$

(viii) $x + \frac{5}{y} = 7$
$3x - \frac{10}{y} = -4$

(ix) $4(x - y) = 3(y - 1)$ (x) $p(x + y) = q(x - y)$
$4x - 5y = 7(x - 7) = 2pq.$

4. Solve the following by the elimination method :

(i) $3p + 2q = \frac{13}{6}$
$6p - 3q = 2$

(ii) $7x - 3y = 31$
$9x - 5y = 41$

(iii) $3x - 4y = -1$
$5x + 2y = 7$

(iv) $\frac{3m - 2}{m + n} = \frac{6}{7}$
$\frac{m}{4} = \frac{n}{3}$

(v) $x + y = 15$
$x + 5 = \frac{3}{2}(y + 5)$

(vi) $\frac{x}{3} + \frac{y}{2} = 1$
$x + 3y = 0$

(vii) $5x - \frac{1}{y} = 3$
$2x + \frac{2}{y} = 5$

(viii) $x + 5y = 36$
$\frac{x + y}{x - y} = \frac{5}{3}$

(ix) $2x - 3y + 10 = 0$
$\frac{x}{2} + \frac{y}{3} - 4 = 0$

(x) $\frac{x + a}{a} = \frac{y + b}{b}$
$ax - by = a^2 - b^2$

(xi) $ax + by = 1$
$bx + ay = \frac{(a + b)^2}{a^2 + b^2} - 1.$

5. Solve the following by the cross-multiplication method :

(i) $3x - 5y = 7$
$8y = 5x - 12$

(ii) $2y - 7z + 18 = 0$
$3z - 5y + 4 = 0$

(iii) $3x + 4y = 25$ (iv) $4l + 3m = 20$

$2x - 3y = -6$ $6l - m = 8$

(v) $2u + v = 13$ (vi) $6y - x = 1$

$5u - 3v = 16$ $\frac{x + y}{x - y} = \frac{3}{2}$

(vii) $\frac{x}{2} + \frac{y}{3} = 1$ (viii) $2x - \frac{3}{y} = 9$

$\frac{x}{3} + \frac{y}{2} = 1$ $3x + \frac{7}{y} = 2$

(ix) $\frac{3}{x} + 7y + 12\frac{1}{2} = 0$ (x) $x + 2y = 3 = 4x - y$

$\frac{14}{x} + 3y - 1 = 0$

(xi) $a(x + y) - b(x - y) = 2a$

$(a^2 - b^2)(x - y) = 4ab.$

6. Solve the following (use any method) :

[If division by x, y or xy is necessary, assume $x \neq 0$, $y \neq 0$ or $x \neq 0$ and $y \neq 0$ respectively.]

(i) $4x + 5y = 20 = 14x - \frac{15y}{2}$ (ii) $\frac{x}{7} = \frac{y}{8}$, $17x - 14y = 7$

(iii) $19x + 17y = 13$

$17x + 19y = 23.$

[Hint : The equations may be solved by addition and subtraction of the two given equations.]

(iv) $2x + \frac{5}{y} = 29$ (v) $\frac{3}{x} + 7y = \frac{5}{2}$

$8x - \frac{3}{y} = 1$ $\frac{1}{4x} - \frac{y}{5} = \frac{1}{80}$

(vi) $\frac{1}{x} + \frac{1}{y} = 5$ (vii) $\frac{2}{x} + \frac{5}{y} = 7$

$\frac{2}{x} - \frac{5}{y} = 1$ $\frac{7}{x} - \frac{2}{y} = 5$

(viii) $\frac{x + y}{xy} = 16, \frac{x - y}{xy} = 2$

[Hint : $\frac{x + y}{xy} = \frac{1}{y} + \frac{1}{x}, \ \frac{x - y}{xy} = \frac{1}{y} - \frac{1}{x}$.]

(ix) $\frac{4x + 3y}{xy} = 6, \frac{3x + 4y}{xy} = 1$

(x) $x + y = 4xy, x - y = \frac{2}{3} xy, x \pm 0, y \pm 0$

[Hint : Divide both sides by xy.]

(xi) $x + y = 2xy, x - y = xy, x \pm 0, y \pm 0$

(xii) $3x - 2y = xy, 5x + 6y = 11xy$

(xiii) $\frac{xy}{x + y} = 2, \frac{xy}{x - y} = 6$

[Hint : $\frac{x + y}{xy} = \frac{1}{2}, \frac{x - y}{xy} = \frac{1}{6}$]

(xiv) $\frac{28xy}{4x + 7y} = \frac{1}{2}, \frac{28xy}{16x - 21y} = 1$

(xv) $\frac{3}{x + y} + \frac{5}{x - y} = 6, \frac{8}{x + y} - \frac{3}{x - y} = -\frac{1}{3}$

[Hint : put $x + y = u, x - y = v$.]

(xvi) $\frac{3}{x + y} + \frac{4}{x - y} = 5, \frac{2}{x + y} + \frac{5}{x - y} = 5\frac{2}{3}$.

(xvii) $(2x + 3y - 7)^2 + (5x - 4y - 6)^2 = 0$

[If $a^2 + b^2 = 0$, then $a = 0$ and $b = 0$]

(xviii) $2x^2 - 2xy + y^2 - 4x - 4 = 0$

[Hint : Express the left hand side as the sum of two squares.]

7. Solve the equations $8x - 11y = 5$ and $3x + 4y = 10$. Put $x = u$, $y = v$ and solve the equations. Is there any difference in the result ?

8. State in which case of the following simultaneous equations, you will get only one solution, an infinite number of solutions and solution.

(i) $3x + 5y = 7$
$8x - 2y = 2$

(ii) $3x - 2y = 8$
$9x - 6y = 7$

(iii) $7x - 11y = 10$
$2x - 7y = 5$

(iv) $8x - 2y = 7$
$4x - y = \frac{7}{2}$

(v) $x + 2y = 8$
$x + 2y = 15$

Miscellaneous Exercise 4

Work out the following problems after expressing them in mathematical language.

1. A number of two digits is seven times the sum of the digits. If 18 be subtracted from the number, the digits interchange their places. Find the number.

2. A number consists of two digits. Adding 1 to the digit in the unit's place and subtracting 1 from that in the ten's place, the digits become the same. The number obtained by interchanging digits bears a ratio 19 : 25 with the original number. Find the number.

3. A number between 10 and 100 is 5 times the sum of its digits ; if 9 be added to it, the digits interchange their places. Find the number.

4. A fraction becomes $\frac{1}{3}$ when 1 is added to the denominator and $\frac{1}{2}$ when 1 added to the numerator. Find the fraction.

5. The fraction formed by taking the difference of the numerator from the denominator of a fraction as numerator and the sum of the numerator and denominator as denominator is $\frac{1}{6}$. The number obtained on subtracting 1 from the numerator is half the number formed on adding 1 to the denominator. Find the fraction.

6. The sum of the numerator and denominator of a fraction is 7. When 1 is added to the numerator, it is equal to the denominator. Find the fraction.

7. The present age of father is twice that of his son. 14 years ago the father was three times as old as his son ; find their present ages.

8. 5 litres of milk and 4 litres of kerosene cost Rs. 28, while 4 litres of milk and 5 litres of kerosene cost Rs. 26. If a person buys 1 litre of milk and 1 litre of kerosene, find the price he has to pay for.

9. 5 pencils and 2 pens cost Rs. 9 and 3 pens and 4 pencils cost Rs. 10 ; find the difference between the prices of a pencil and a pen.

10. 8 quires of paper and 10 bound exercise books cost Rs. 50, whereas 5 quires of paper and 6 bound exercise books cost Rs. 30·50. Find separately the price of 1 quire of paper and 1 bound exercise book.

11. Kajari tells Sarbari, "If you give me Rs. 2, my money will be twice that of you." "If you give me Re. 1", says Sarbari, "my amount will be equal to yours." How much does each of them have ?

12. Laikat and Irfan are mango-seller. If Irfan takes 30 mangoes from Laikat, the number of mangoes with him is twice the number of mangoes remaining with Liakat. Again if Laikat takes 10 mangoes from Irfan, the number of mangoes with him is three times the number of mangoes remaining with Irfan. How many mangoes are there with each ?

13. Suresh and Majid can do a piece of work in 12 days. Suresh does work for 16 days and the rest is finished by Majid alone. The work is comopleted in 22 days. How long would Suresh take to do the work on his own ?

14. One of the two workers does work for 16 days and the other for 15 days. The wage of 4 days of the first is less than the wage for 7 days of the second by Re 1. Rs. 125 is paid to them. What are their daily wages ?

15. A and B, 27 Km apart, walking in the same direction meet in 9 hours ; but if they had walked in opposite directions, they would have met in 3 hours. If the speed of *A* be greater than that of *B*, find their respective speeds.

16. A train moving with uniform speed for a certain distance takes 6 hours less if its speed be increased by 6 Kilometres per hour ; it would have taken hours more, had its speed been diminished by 4 kilometres per hour. Find the distance of the journey and the speed of the train.

17. A man rows 24 Km downstream and 12 Km upstream in 10 hours : he also rows 36 Km downstream and 16 Km upstream in 14 hours. Find the rate of rowing in still water and of the current.

18. A and B are on the banks of a river 55 Km apart. The river flows from A to B. C is between A and B and is at a distance of 22 Km from A. A boat takes 14 hours in going from C to B and then to return to B. If the speeds of the boats in still water be same, find the speed of the boats and that of the river.

19. In a mixture of liquids the first and the second are in in the ratio 2 : 3. In another, they are in the ratio 5 : 4. In what ratio these two mixtures would have to be taken to form a mixture so that the two liquids would be in equal quantities in that mixture.

20. A man has a fixed monthly income ; if the expenses in February were less than those in July by Rs. 50, the ratio of expenses to the savings would be 5 : 3 ; if the expenses in March were more than those in January by Rs. 25, the ratio of expenses to the savings would be 11 : 3. Find his monthly income.

21. The total number of students in two schools is 990. If the number of students in one school decreases by 5% and increases in the other by 14%, the number of students in the two schools would be equal. How many students are there in each school ?

22. The cost of three cycles and two mopeds is Rs. 7,800. If tax be levid at the rate of 10% and 25% on cycle and moped respectively, the total price would be Rs. 9,480. What is the price of a cycle and a moped separately ?

23. If the length of a rectangular field is increased by 5 metres and the breadth is decreased by 3 metres the area of the field is decreased by 35 Sq. metres. Again if the length is decreased by 10 metres and the breadth is increased by 10 metres, the field is converted into a square. Find the dimensions of the rectangular field.

24. If the length of a rectangular field is decreased by 2 metres and breadth is increased by 3 metres, the area is increased by 24 Sq. metres. Again if the length decreased by 10 metres and the breadth. Again if the length is increased by 2 metres and the breadth is decreased by 2 metres, the area is decreased is 16 Sq. metres increased by 10 metres, the field is converted into a square. Find the dimensions of the rectangular field.

25. A path of unifrom width surrounds a circular garden. The sum of the circumferences of inner and outer circles is 176 metres. If the path be 6 metres wide, what is the radius of the garden ?

Miscellaneous Exercise 5

1. In which quadrant, or on which axes and in which direction do the following points lie ? Plot the points on the graph paper.

(i) (– 4, 0), (ii) (5,0), (iii) (0, – 6), (iv) (6, 4), (v) (–2, 7), (vi) (5, –8), (vii) (–9, –9), (viii) (0, 7).

2. Plot the following pairs of points and connect each pair by a straight line.

(i) (5, 2), (10, 15) ; (ii) (3, 4), (–6, 9) ; (iii) (5, 0), (–5, –5) ; (iv) (–2, 6), (7, –5) ; (v) (10, 0), (0, –10) ; (vi) (0, 0), (3, –8) ; (vii) (8, 2), (–7, 2) ; (viii) (–2, 8), (–2, 6) ; (ix) (5, –4), (11, –4) ; (x) (7, 2), (7, –7)

3. Plot the following points, and examine if they are collinear :

(i) (0, –3), (–3, –8), (6, 7) (ii) (2, 1), (–5, 8), (2, 6) (iii) (2, 3), (–6, 5), (6, 2) (iv) (1, 2), (–2, 3), (5, 4)

4. Draw the graphs of the following equations :

(i) $3x - 4y = 0$, (ii) $y = -6$, (iii) $x = 5$, (iv) $2x + 3y = 5$, (v) $\frac{x}{3} - \frac{y}{2} = 1$ (vi) $2x + 3y = 12$, (vii) $2x - 5y = 10$,

(viii) $\frac{x + y}{x - y} = \frac{5}{3}$, (ix) $4x + 9 = 5y - 14$.

5. (i) Draw the graph of $\frac{x + 2}{3}$. From the graph, find the value of the expression when $x = -2$. Also find, for waht value of x, the value of the expression is 3.

[*Hint* : Draw the graph of $y = \frac{x + 2}{3}$. From the graph, find the y co-ordinate of the point of which the x co-ordinate is – 2, and find x-co-ordinate of the point of which the y co-ordinate is 3.]

(ii) Draw the graph of $\frac{2x-3}{5}$. From the graph, find the value of the expression when $x = 4$; find also the value of x for which the value of the expression is – 1.

(iii) Taking the same axes of co-ordinates and using same unit, draw the graphs of $\frac{3x-1}{2}$ and $\frac{2x+6}{3}$. Find for what value of x the two expressions will have the same value.

[Hint : The x co-ordinate of the point of intersection where the two graphs intersect is the required value.

Remark : Note that the value of x is the solution of the equation $\frac{3x-1}{2} = \frac{2x+6}{3}$. Thus the two equations are solved graphically.]

6. Solve graphically :

(i) $3(1 - 2x) = 7x + 29$, (ii) $3(2x + 5) = 5x + 18$,

(iii) $\frac{12 - 5x}{6} = \frac{78 - x}{12}$ (iv) $\frac{3x + 36}{10} = \frac{50 - x}{7}$.

7. (i) Draw the graph of the equation $3x - 5y = 8$. Find the value of y, when $x = -4$ and the value of x, when $y = 5$ that will satisfy the equation.

(ii) Draw the graph of the equation $\frac{x}{4} + \frac{y}{3} = 1$. Find the length of the portion of the graph intercepted between the axes.

[*Hint* : With origin as centre and radius equal to the intercept cut off a portion from the x- or y-axis. Count the number of units.]

(iii) Draw the graph of the equation $3x - 2y = 24$. Find the area of the triangle formed by the graph and the co-ordinate axes.

[*Hint* : The area of a right-angled triangle is half of the product of the sides containing the right angle.]

(iv) Draw the graph of the equation $\frac{x}{3} + \frac{y}{5} = 1$. Find the area of the triangle formed by the graph with the co-ordinate axes.

(v) Draw the graph of the equation $12x + 5y = 60$. Find the hypotenuse and the area of the right-angled triangle formed by the graph and the co-ordinate axes.

(vi) Draw the graphs of the equations $y = 3$, $2x - y - 1 = 0$ and $4x + 3y - 37 = 0$. Find the co-ordinates of the vertices of the triangle formed by the above graphs. Also find the area of the triangle.

(vii) Draw the graphs of the equations $x = -3$, $7x + 6y = 21$ and $2x - 3y - 6 = 0$. Find the co-ordinates of the vertices and the area of the triangle formed by the graphs.

(viii) Draw the graphs of the equations $x + y = 7$, $x - y + 3 = 0$ and $5x + y - 27 = 0$. Find the area of the triangle formed by the graphs.

[*Hint* : The vertices of the triangle are (5, 2), (4, 7) and (2, 5). Through (2, 5) and (4, 7) draw straight lines parallel to x-axis and through (2, 5) and (5, 2) draw straight lines parallel to y-axis. From the area of the rectangle thus formed subtract the areas of the right-angled triangles to obtain the area of the triangle.]

8. Draw the graphs of the following simultaneous equations and find their points of intersection.

(i) $x = 5, 2x - 5y = 5$; (ii) $y = -3, 3x + 2y + 3 = 0$

(iii) $3y - 2x + 10 = 0, y = 4x$

(iv) $\frac{x}{5} + \frac{y}{2} = 1, x - 4y + 21 = 0$

(v) $3x - 7y = 37, \frac{x}{3} + \frac{y}{4} = 0$

(vi) $8x + 5y = 11, 3x - 4y = 10$

[Note that the co-ordinates of the point of intersection satisfy both the equations, so the x co-ordinate and the y co-ordinate of that point constitute the solution of the equations.

9. Solve graphically :

(i) $x = 5, x + y = 6$; (ii) $y = -6, 2x + 3y = 10$; (iii) $4x - 5y = 7, 2x + 3y = 9$; (iv) $x - 5y = 6, 3x - 7y = 2$; (v) $5x - 2y = 1, 3x + 5y - 13 = 0$; (vi) $x + 5y + 6 = 0, 4x - 3y - 22 = 0$, (vii) $x + y = 0, 8x - 5y + 13 = 0$; (viii) $x - 2y - 3 = 0, \frac{x}{7} + \frac{y}{5} + 2 = 0$.

10. Draw the graphs of the equations $3x + 4y = -7$ and $3x + 4y - 1 = 0$. What is your observation regarding the two graphs ?

11. Draw the graphs of the equations $2x + 7y - 11 = 0$ and $7x - 2y - 5 = 0$. Measure the angle between the graphs with a protractor.

12. Draw the graphs of the equations $2x + 3y = 5$ and $6x + 9y = 15$. What can you say about the two graphs ?

13. Corresponding to $x = 0, 1, 2$ respectively find the values of y that will satisfy the equation $2y = 6x + 1$. Plot the points by taking the values of x and the corresponding values of y as co-ordinates and by taking the length of two sides of the smallest square as the unit and hence draw the graph of the equation $2y = 6x + 1$. Do you get an integral value of y corresponding to an integral value of x satisfying the equation ?

[*Remark* : If the terms of an equation be divided by their common factor and if in the resulting equation the co-efficient of x or y be a factor of the other, the equation will have no solution as integral values of both x and y.]

Miscellaneous Exercise 6

1. If $a : b : c = 2 : 3 : 4$ and $2a - 3b + 4c = 33$, find the values of a, b and c.

2. If $3x - 2y : 2x - y = 4 : 3$ find the values of $x : y$.

3. If $a : b = c : d$, prove that :

   (i) $a^2 + c^2 : b^2 + d^2 = ac : bd$

   (ii) $(a + c)^3 ; (b + d)^3 = a(a - c)^2 : b(b - d)^2$

   (iii) $\dfrac{(a - b)\,(a - c)}{a} = (a + d) - (b + c)$

   (iv) $\dfrac{a^2 + c^2}{a + c} + \dfrac{b^2 + d^2}{b + d} = \dfrac{(a + b)^2 + (c + d)^2}{a + b + c + d}$

4. If p, q, r, s are in continued proportion, prove that :

   (i) $(pq + qr + r^2) : (qr + rs + s^2)$
   $= (p^2 + q^2 + r^2) : (q^2 + r^2 + s^2)$

   (ii) $\dfrac{q}{s} = \dfrac{p^2+q^2+r^2}{q^2+r^2+s^2}$

5. (i) If $a : b = c : d\ 2{\cdot}5 : 1{\cdot}5$, find the value of ad : bc and $a + c : b + d$

   (ii) If $\dfrac{a}{3} = \dfrac{b}{4} = \dfrac{c}{7}$, show that $\dfrac{a+b+c}{c} = 2$

6. If $\dfrac{a}{q - r} = \dfrac{b}{r - p} = \dfrac{c}{p - q}$, show that $a+b+c = 0 = pa+qb+rc$

7. If $x : a = y : b = z : c$, show that

   (i) $\dfrac{x^3+y^3+z^3}{a^3+b^3+c^3} = \dfrac{xyz}{a\,b\,c}$

   (ii) $\dfrac{x^3}{a^3} + \dfrac{y^3}{b^3} + \dfrac{z^3}{c^3} = \dfrac{3xyz}{a\,b\,c}$

   (iii) $(a^2 + b^2 + c^2)\,(x^2 + y^2 + z^2) = (ax + by + cz)^2$

   (iv) $\dfrac{3x^2+5y^2+4z^2}{3a^2+5b^2+4c^2} = \dfrac{x^2+y^2+z^2}{a^2+b^2+c^2}$

8. If $a : b = c : d = e : f$, show that each of them is equal to $\dfrac{pa + qc + re.}{pb + qd + rf}$

9. If $\dfrac{ax+by}{a} = \dfrac{bx-ay}{b}$, show that each of them is equal to x.

10. If $\dfrac{by+cz}{b^2+c^2} = \dfrac{cz+ax}{c^2+a^2} = \dfrac{ax+by}{a^2+b^2}$, show that $\dfrac{x}{a} = \dfrac{y}{b} = \dfrac{z}{c}$

11. If $\frac{ay-bx}{c}=\frac{cx-a^z}{b}=\frac{bz-cy}{a}$, prove that $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$.

12. If $\frac{a}{b+c}=\frac{b}{c+a}=\frac{c}{a+b}$, prove the each of them is equal to $\frac{1}{2}$, or $a+b+c=0$

13. If $\frac{a}{y+z}=\frac{b}{z+x}=\frac{c}{x+y}$, prove that
$\frac{a(b-c)}{y^2+z^2}=\frac{b(c-a)}{z^2-x^2}=\frac{c(a-b)}{x^2-y^2}$

14. If $\frac{x+y}{3a-b}=\frac{y+z}{3b-a}$, show that
$\frac{x+y+z}{a+b+c}=\frac{ax+by+cz}{a^2+b^2+c^2}$

15. (i) If $\frac{a^2}{b+c}=\frac{b^2}{c+a}=\frac{c^2}{a+b}=1$, show that
$\frac{1}{1+a}+\frac{1}{1+b}+\frac{1}{1+c}=1$

(ii) If $\frac{x^2}{by+cz}=\frac{y^2}{ax+by}=\frac{z^2}{ax+by}=1$, show that
$\frac{a}{a+x}+\frac{b}{b+y}+\frac{c}{c+z}=1$

16. If $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$, whow that
$\frac{x^2+y^2}{a^2+bc}=\frac{y^2+xz}{b^2+ca}=\frac{z^2-xy}{c^2-ab}$

17. If $\frac{x^2-yz}{a}=\frac{y^2-zx}{b}=\frac{z^2-xy}{c}$, prove that
$(a+b+c)(x+y+z)=(ax+by+cz)$.

18. If $(x+z):(y+z)=\left(\frac{x}{y}+2\right):\left(\frac{y}{x}+2\right)$, show that
$(x-z):(y-z)=x^2:y^2$

19. If $\frac{x+y}{x-y}=\frac{u}{v}$, show that $\frac{x^2+xy}{xy-y^2}=\frac{u^2+uv}{uv-v^2}$

20. If $\frac{x+2y+7}{5}=\frac{2x+y+11}{7}=\frac{x+y-4}{2}$, show that
$\frac{x+15}{5}=\frac{y+11}{3}=5$.

Hence solve the equations.

Now plotting that points (0, 0), (1, 2·5) and drawing a straight line through these points, we get the number-price graph of the exercise books (fig.15).

To find the price of 6 exercise books, draw a straight line perpendicular to x-axis at a distance 6 units from the origin. The perpendicular meets the graph at the point P. Draw PM perpendicular to y-axis. It is seen that $OM = 15$. Thus, the price of 6 exercise books is Rs. 15·00.

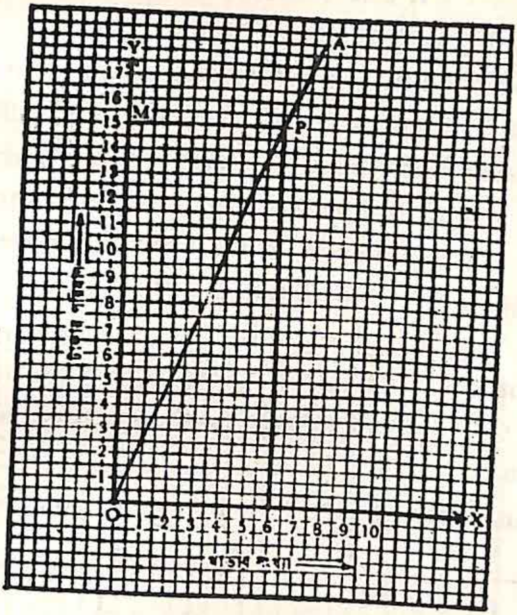


Fig. 15

[*Remark* : As the price of an exercise book or its number cannot be negative, the graph has been drawn only in the first quadrant.]

Example 2 : At 10 A.M. Koushik starts from a certain place at the rate of 4 km an hour. Two hours later, Karim follows Koushik from the same place at the rate of 5 kilometres per hour towards Koushik. Find graphically when and where they will meet.

Let us represent time along x-axis and distance along y-axis. Let the time be reckoned from the instant Koushik starts.

We get the following table for the journey of Koushik :

x (time in hours)	0	1
y (distance in km)	0	4

For the journey of Karim the table may be easily constructed as follows.

x (time in hours)	2	3
y (distance in km)	0	5

Two graphs are drawn using the above tables, with the following units.

Side of one smallest square along x-axis = 1 hour

Side of one smallest square along y-axis = 2 km.

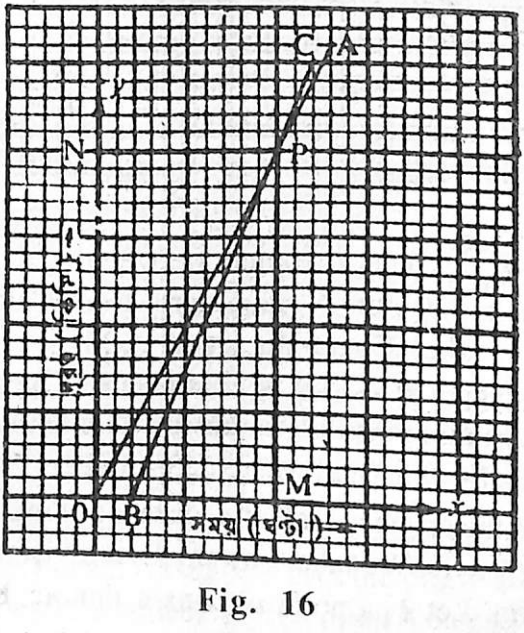
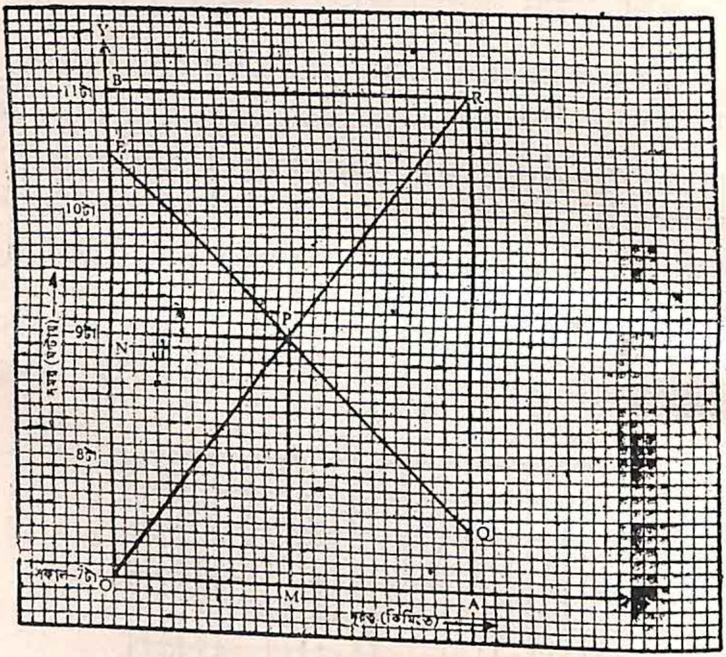


Fig. 16

The graphs intersect at the point P.

The abscissa of $P = 10$ units and the ordinate of $P = 40$ units.

So, Karim will meet Koushik after 10 hours from the insant Koushik started and a distance of 40 km from the starting point.



Example 3. The distance between Calcutta and Kharagpur is 120 km. A train starts from Calcutta at 7 A.M. and reaches Kharagpur at 11 A.M.. another train leaves Kharagpur at 7-30 A.M. and reaches Calcutta at 10-30 A.M. Assuming that two trains run with uniform speed, find graphically when and where they meet.

We take the origin at Calcutta, distance along x-axis and time along y-axis. We take, one smallest division along x-axis = 4 km. Let A, a point on x-axis denote Khargpur, $\therefore OA = 120$ km.

The time at the origin is 7 A.M. Let 10 divisions along y-axis represent 1 hour. Time at *B* is 11 A.M. From *A* and *B* draw perprndiculars to *x*-axis and *y*-axis respectivley. These two perpendiculars meet at the point *R*.

OR respresents the motion-graph of the train bound for Kharagpur. Let the point *Q* on the straight line perpendicular to *x*-axis through the point *A* denote the time 7-30 A.M. and the point *E* on the *y*-axis denote the time 10-30 A.M. Then at 7-30 A.M. the Calcutta bound train starts from the point *Q* and moves along *QE*. *QE* represents the motion-graph of the second train.

The two graphs intersect at the point *P*. From the point *P*, prependiculars *PM* and *PN* are drawn to *x*-axis and *y*-axis respectively.

It is seen from the graph $OM = 15$ divisions, which denote 2 hours

and $ON = 2$ divisions, which denote 60 km.

Thus after 2 hours from 7 A.M. i.e., at 9 A.M. the two trains meet at a distance 60 km from Calcutta.

[*Remarks* : the units along x-axis and *y*-axis are to be chosen initially so that the graph can be drawn conveniently according to the given conditions of the problem.]

Example 4: The distance between Kolaghat and Panskura is 24 km. A man started from Kolaghat at 6 A.M. at a rate of 3 km/hour towards Panskura. After walking 2 hours he took rest for an hour and then he continued as before. Another man started from Panskura at 8 A.M. at a rate of 4 km/hour towards Kolaghat. When and where did they meet ?

Suppose, we take distance (in Km) along *x*-axis and time (in hour) along *y*-axis. We take the origin at Kolaghat and according to a suitable unit a point *A* on *x*-axis denote Panskura. Let the time at the origin be 6 A.M.

The first man after moving for 2 hours at the rate of 3 km/hour arrives at a point *B* and takes rest for an hour there. During this period his position is not changed but one hour has elapsed there. So after one hour, his position in the graph will be at the point *C*. Join *O*, *B* and *B*, *C*. After this the man continues to move in the direction *CD* at a rate of 3 km/hour. The motion-graph of the first is thus *OBCD*.

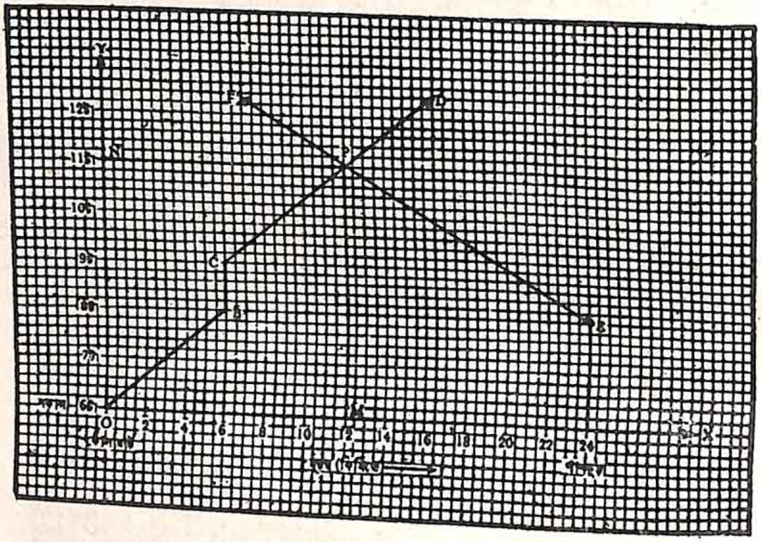


Fig. 18

The second man starts from Pansukara at 8 A.M. at a rate of 4 km/hour towards Kolaghat along *EF*. The two graphs meet at the point *P*. From the point *P*, perpendiculars *PM* and *PN* are drawn to *x*-axis and *y*-axis respectively. It is seen from the graph that *OM* = 12 units and *ON* = 5 units.

Thus after 5 hours from 6 A.M. i.e., at 11 A.M. two men meet each other at a distance 12 km from Panskura or Kolaghat.

Exercise 20

1. If 5 kilogrames of tea cost Rs. 175, find graphically the price of 20 kilograms of tea.

2. If 1 litre of Kerosene costs Rs. 1·90, find graphically the price of 8 litres of Kerosene oil. How many litres of Kerosene may be obtained for Rs. 28·50 ?

3. In the following table, the body temperatures of a patient are shown at different hours of day :

House of the day	6 A.M	10 A.M	12 Noon	2 P.M.	4 P.M.	6 P.M.	8 P.M.
Temp. (celsius)	38	37	37·5	38	38·5	39	38

From the above data draw a graph to find the temeperature of the patient at (i) 7 A.M. and (ii) 7 P.M.. (It is assumed that during any interval the temperature of the patient has changed uniformly.)

4. A man travels 4 km in an hour; find graphically how far the man goes in 30 min. and what time he takes in travelling 22 km.

5. If 4 lemons cost Rs. 1·60, find graphically the price of 14 lemons and also find the number of lemons that will be obtained for Rs. 10.

6. The population of a country is shown in the following table :

Year	1930	1940	1950	1960	1970
Population (in thousand)	15	20	25	30	35

If the population increases at a uniform rate, find graphically the same in 1990 and in 2020.

7. Sagarpur and Keshpur are two village 28 km aprt. From those two villages, two villagers start at the same time towards each other with uniform speed of 4 km/hour and 3 km/hour respectively. Find graphically when and where they meet.

8. Abani starts at 10 A.M. with a uniform speed of 5 km/hour. Two hours later Biswajit follows him by a bicycle with a uniform

speed of 7 km/hour. Find graphically when and where they meet each other.

9. A train starts from Calcutta at 9 P.M. and reaches a station 72 km away at 11 P.M.; If the two trains do not stop anywhere after they have started, find graphically when and where the trains meet each other.

10. The salary of a worker increases annually at a fixed rate. If his salary becomes Rs. 600 and Rs. 740 after 5 years and 12 years of service respectively, find graphically his initial salary and the salary he would have after 15 years.

11. Ramnagar and Habibpur are 27 km apart. Biren starts from Ramnagar at 9 A.M. with a uniform speed of 3 km/hours towards Habibpur. After walking 2 hours he takes rest for 1 hour 30 min. and then continues walking as before. Majid starts from Habibpur at 9 A.M. with a uniform speed of 4 km/hour towards Ramnagar; find graphically when and where they meet each other on the way.

CHAPTER 8

Quadratic Equation

You know how to form an equation to solve it. The equations, that you have solved, involve unknown quantities of first degree only. So, these equations are called equations of the first degree. Thus,

(1) $3(5x-8) = 2(7-2x)$ is an equation of first degree, in one unknown.

(2) $x + 3y = 19$

$4x + y = 14$

are simultaneous equations of first degree in two unknowns.

But the conditions of a problem may be such that the power of the unknown quantity in the equations so formed is greater than one. We shall now discuss the equas into which the highest power of the unknown quantity is two. Let us see in how these equations are formed.

Suppose, in a school each student of a particular class subscribes 10 P. more than the number of students in the class. If Rs. 20 be collected in all.

If this statement is to be expressed in mathematical language let us assume the number of students be x.

∴ Each student subscibes $(x+10)$ P.

Toal collection is Rs. 20 or 2000 P.

∴ By the given condition, $x(x+10) = 2000$

or, $x^2 + 10x = 2000$.............. (1)

Let us consider another example.

A number is less than its square by 20 .

Suppose, the number be x.

From the given conditions we will get, $x = x^2 - 20$ .... (2)

Equations (1) and (2) in one unknown and in each, the highest power of x is 2. Such equations are called quadratic equations in one unknown.

Further examples of quadratic equations are :

(a) $5x^2 - 8x + 2 = 0$ (b) $x^2 = 4$

(c) $y + \frac{1}{y} = 6$ (d) $(x-2)(x+3) + 5 = 0$

[*Remarks* : (1) To know the highest power of the unknown quantity in an equation, it is sometimes necessary to simplify the terms of an equation. Thus, the highest power of x in the equation $\frac{x}{3} + \frac{3}{x} = \frac{17}{4}$ may appear to be one. But on simplification we get $4x^2 - 51x + 36 = 0$.

So, it is a quadratic equation.

Similarly, the equation $\frac{(x+1)^2 - (x-1)^2}{(x+1)^2 - (x-1)^2} = 2$ is really a quadratic equation, for, on simplification there is no terms containing x^3.

Again, the equation $3(2x^2 - 5x + 2) = 2(3x^2 - 8x + 4)$, though appeared to be a quadratic equation, is really a linear equation.

(2) In the equation $x^4 - 5x^2 + 6 = 0$, the highest power of x is 4; if we put $x^2 = y$; shall get $y^2 - 5y + 6 = 0$. So $x^4 - 5x^2 + 6 = 0$ can be reduced to quadratic equation in x^2.
Similarly $x^6 - x^3 - 2 = 0$ is a quadratic equation in x^3.

In practice, in case of the solution of problems, you must have seen that, we have to form an equation first. The same applies to quadratic equations also. Form equations from the following exercise as shown previously.

Exercise 22

Express the following statements in mathematical language (in the form of equation) :

1. 42 is divided into two parts-such that one is equal to the square of the other.
2. The product of two consecutive odd positive numbers is 143.
3. The sum of the squares of two consecutive number is 313

4. A man bought some kilograms of sugar for Rs. 80. Had he got 4 kg more at the same price, the price of sugar per kg would have been less by Re. 1.

5. The length of the diagonal of a rectangle is 15 m and one of its sides is longer than another by 3 m.

6. Two stations A and B are 300 km apart. A train goes from the station A to B at a uniform speed. Had the speed of the train been 5 km/hour more, it would have taken 2 hours less to travel the same distance.

7. A clock dealer bought a clock and sold it for Rs. 336. He gained as much in percentage as was his cost price.

8. When the speed of the current is 2 km/hour, a boatman takes 5 hours to go 14 kilometres downstream and to return.

9. Mahim takes 3 hours more to do a piece of work than Majid. They together complete it is 2 hours.

10. A number consists of two digits. The digit in the init's place exceeds that in the ten's place by 6. The product of the digits is less than the number by 12.

11. A uniformly wide path surrounding a rectangular play ground, 45 m broad 40 m long and has area of 450 sq.m.

12. The speed of current is 4 km/hour. A boat takes 3 hours to go 18 km downstream and return 6 km upstream.

Some points about quadratic equation :

The general form of a quadratic equation is

$ax^2 + bx + c = 0$ .... (A)

Here, a = co-efficient of x^2

b = co-efficient of x

c = constant term or term independent of x.

Here, $a \neq 0$, otherwise the equation will no longer be a quadratic equation.

When $b \neq 0, c \neq 0$, i.e., when the term containing x and the term independent of x are both present, the quadratic equation is called as adfected quandratic equation.

Thus $5^2 + 6x - 2 = 0$ is an adfected quadratic equation.

Again when $b = 0, c \neq 0$ i.e., when he term containing x is absent, the quadratic equation is called a pure quadratic equation. $x^2 - 100 = 0$ is a pure quadratic equation.

If any quadratic equation (on simplification, if necessary) is written in the form $ax^2+bx+c = 0$ it will be observed that the left hand side is a quadratic empresion.

For example, the equation, $x^2 - 5x = -6$ can be written as

$x^2 - 5x + 6 = 0$ .... (1)

The left hand side of (1), i.e., $x^2 - 5x + 6$ is a quadratic expression. You know how to factorise such an expresion.

For example, $x^2 - 5x + 6 = x^2 - 3x - 2x + 6$

$= x(x - 3) - 2(x - 2)$

$(x - 3)(x - 2)$

Now, when $ab = 0$, either 1 $a = 0$, or $b = 0$, or, 2 $a = 0$ and, $b = 0$, but, it can never be that neither of a and b is zero.

From, $(x-2)(x-3) = 0$, we can write

either, $x - 2 = 0$

Or, $x - 3 = 0$

Thus, by factorising the left hand side of the equation $x^2-5x + 6 = 0$, we can write

$(x-2)(x-3) = 0$

Therefore, either $x - 2 = 0$ .... (1)

Or, $x - 3 = 0$ .... (2)

(1) and (2) are two linear equations. You know how to solve them.

From (1) we get $x = 2$ and from (2) we get $x = 3$.

Thus, $x = 2$ and $x = 3$ are solutions of the equation $x^2 - 5x + 6 = 0$

Examine what happens by putting $x = 2$ and $x = 3$ separately in the equation $x^2 - 5x + 6 = 0$.

In the light of the above discussion we may proceed to solve a quadratic equation in the following way :

(1) First, the equation (simplifying, if necessary) is to be written in the form $ax^2 + bx + c = 0$ then,

(2) l.h.s. of the equation is factorised, and

(3) the two factors thus obtained are put equal to zero.

The resulting linear equations, on solving, give the solutions of the given quadratic equation.

The solutions thus ontained are called the roots of the given quadratic equation. You will learn later on, that a quadratic equation cannot have more than two roots.

[Remark : (1) When $b \neq 0$, $c = 0$, one root of the quadratic equation is always zero. Thus, the equation $5x^2 + 3x = 0$ has no constant term. We can write it as $x(5x + 3) = 0$

$\therefore$ either $x = 0$, or, $5x + 3 = 0$

$5x + 3 = 0$ gives, $x = -\frac{3}{5}$

Thus, the roots of the equation,

$5x^3 + 3x = 0$ are $0, -\frac{3}{5}$

(2) When $b = 0$, $c = 0$, both the roots of the quadratic equation are zero. Thus, for the equation $3x^2 = 0$,

we have, $x^2 = 0$

or, $x = 0, 0$.]

Example 1. Solve : $x^2 = 16$

$x^2 = 16$

or, $x^2 - 16 = 0$

or, $(x+4)(x-4) = 0$

Therefore, one of the expressions $x+4$ and $x-4$ must be zero.

When $x+4 = 0, x = -4$

and when $x-4 = 0, x = 4$

Thus, the required solutions are $x = \pm 4$.

[*Remark* : We may solve a pure quadratic equation by the following alternative method.

Keeping the unknown quantity in the left had side and the constan term in the right hand side and by taking square root of both sides, the equation can be solved.

Thus in the case of the equation $x^2 = 16$ we get,

$x^2 = 16$

or, $x = \pm 4$]

Example : 2 Solve : $3x^2 - 10x + 3 = 0$

$3x^2 - 10x + 3 = 0$

or, $3x^2 - 9x - x + 3 = 0$

or, $3x(x-3) - (x-3) = 0$

or, $(x-3)(3x-1) = 0$

Therefore, one of the expressions $x-3$ and $3x-1$ must be zero.

When, $x-3 = 0, x = 3$

and when $3x-1 = 0, x = \frac{1}{3}$

Thus required solutions are $x = 3, \frac{1}{3}$

Example 3 : Solve : $5x^2 - 4x = 12$

$5x^2 - 4x = 12$

or, $5x^2 - 4x - 12 = 0$

$$\text{or, } 5x^2 - 10x + 6x - 12 = 0$$
$$\text{or, } 5x(x-2) + 6(x-2) = 0$$
$$\text{or, } (x-2).(5x+6) = 0$$

Therefore, one of the expressions $x - 2$ and $5x + 6$ must be zero

when $x - 2 = 0, x = 2$

and when $5x + 6 = 0, 5x = -6$

or, $x = -\frac{6}{5}$

$\therefore$ required solutions are $x = 2, -\frac{6}{5}$

Example 4 : Solve : $(x + 4)(2x - 3) = 6$

$$(x + 4)(2x - 3) = 6$$
$$\text{or, } 2x^2 - 3x + 8x - 12 = 6$$
$$\text{or, } 2x^2 + 5x - 18 = 0$$
$$\text{or, } 2x^2 + 9x - 4x - 18 = 0$$
$$\text{or, } x(2x + 9) - 2(2x + 9) = 0$$
$$\text{or, } (2x + 9)(x - 2) = 0$$

Therefore, one of the expressions $2x + 9$ and $x - 2$ must be zero.

When $2x + 9 = 0,\ x = -\frac{9}{2}$

and when $x - 2 = 0, x = 2$

$\therefore$ required solutions are $-\frac{9}{2}, 2$

Example 5 : Solve : $\frac{1}{x+1} + \frac{2}{x+5} = \frac{1}{2}$

$$\frac{1}{x+1} + \frac{2}{x+5} = \frac{1}{2}$$
$$\text{or, } \frac{(x+5)+2(x+1)}{(x+1)(x+5)} = \frac{1}{2}$$
$$\text{or, } \frac{x+5+2x+2}{x^2+6x+5} = \frac{1}{2}$$

or, $\frac{3x+7}{x^2+6x+5} = \frac{1}{2}$

or, $x^2 + 6x + 5 = 6x + 14$

or, $x^2 = 9$

or, $x = \pm 3$, (taking square root of both sides)

Alternative method :

$\frac{1}{x+1} + \frac{2}{x+5} = \frac{1}{2}$ or, $\frac{1}{x+1} = \frac{1}{2} - \frac{2}{x+5}$

or, $\frac{1}{x+1} = \frac{x+5-4}{2(x+5)}$ or, $\frac{1}{x+1} = \frac{x+1}{2x+10}$

or, $x^2 + 2x + 1 = 2x + 10$

or, $x^2 = 9$

$x = \pm 3$ (taking square root of both sides)

Example 6 : Solve : $\frac{x}{3} + \frac{3}{x} = 4\frac{1}{4}$

$\frac{x}{3} + \frac{3}{x} = 4\frac{1}{2}$

or, $\frac{x^2+9}{3x} = \frac{17}{4}$ $[\because x \neq 0]$

or, $4x^2 + 36 = 51x$

or, $4x^2 - 51x + 36 = 0$

or, $4x^2 - 48x - 3x + 36 = 0$

or, $4x(x - 12) - 3(x - 12) = 0$

or, $(x - 12)(4x - 3) = 0$

Therefore, one of the expressions $x - 12$ and $4x - 3$ must be

350.

When $x - 12 = 0$, $x = 12$, when $4x - 3 = 0$, $x = \frac{3}{4}$

$\therefore$ required soluions are, $x = 12, \frac{3}{4}$

Alternative method :

$\frac{x}{3} + \frac{3}{x} = 4\frac{1}{4}$

$$\text{or, } \frac{x}{3} + \frac{3}{x} = 4 + \frac{1}{4}$$

$$\text{or, } \frac{x}{3} - 4 + \frac{3}{x} - \frac{1}{4} = 0$$

$$\text{or, } \frac{x-12}{3} + \frac{12-x}{4x} = 0$$

$$\text{or, } (x - 12)\left(\frac{1}{3} - \frac{1}{4x}\right) = 0$$

$\therefore$ one of the expressions $x - 12$ and $\frac{1}{3} - \frac{1}{4x}$ must be zero

When $x - 12 = 0, x = 12$

When $\frac{1}{3} - \frac{1}{4x} = 0, \frac{1}{3} = \frac{1}{4x}$

or, $4x = 3$

or, $x = \frac{3}{4}$

$\therefore x = 12, \frac{3}{4}$

Example 7 : Solve : $\frac{x-3}{x+3} - \frac{x+3}{x-3} + 6\frac{6}{7} = 0$

Putting $\frac{x-3}{x+3} = a$, the equation can be written as

$$a - \frac{1}{a} + 6\frac{6}{7} = 0$$

$$\text{or, } \frac{a^2-1}{a} = \frac{-48}{7}$$

$$\text{or, } 7a^2 - 7 = -48a$$

$$\text{or, } 7a^2 + 48a - 7 = 0$$

$$\text{or, } 7 + 49a - a - 7 = 0$$

$$\text{or, } (a + 7)(7a - 1) = 0$$

$\therefore$ one of the expressions $a+7$ and $7a-1$ must be zero.

When $a + 7 = 0, a = -7,$

when $7a - 1 = 0, 7a = 1$ or, $a = \frac{1}{7}$

$\therefore a = -7, \frac{1}{7}$

when $a = -7$, we have $\frac{x-3}{x+3} = -7$

or, $x - 3 = -7x - 21$

or, $8x = -18$

or, $x = -\frac{9}{4}$

When $a = \frac{1}{7}$, we have, $\frac{x-3}{x+3} = \frac{1}{7}$

or, $7x - 21 = x+3$

or, $6x = 24$

or, $x = 4$

Thus, required solutions are $x = -\frac{9}{4}, 4$

Alternative method :

$\frac{x-3}{x+3} - \frac{x+3}{x-3} + 6\frac{6}{7} = 0$

Putting $\frac{x-3}{x+3} = a$, the equation can be written as

$a - \frac{1}{a} + 6\frac{6}{7} = 0$

or, $a - \frac{1}{a} + 7 - \frac{1}{7} = 0$

or, $a + 7 - \frac{1}{a} - \frac{1}{7} = 0$

or, $(a + 7) - \frac{a+7}{7a} = 0$

or, $(a + 7)(1 - \frac{1}{7a}) = 0$

$\therefore$ one of the expressions $a + 7$ and $1 - \frac{1}{7a}$ must be zero.

When $a + 7 = 0, a = -7$

when $1 - \frac{1}{7a} = 0, 1 = \frac{1}{7a}$

or, $7a = 1$

or, $a = \frac{1}{7}$

when $a = -7$, we have $\frac{x-3}{x+3} = -7$

or, $x - 3 = -7x - 21$

or, $x + 7x = -21 + 3$

or, $8x = -18$

or, $x = -\frac{9}{4}$

when $a = \frac{1}{7}$, we have $\frac{x-3}{x+3} - \frac{1}{7}$

or, $7x - 21 = x + 3$

or, $6x = 24$

or, $x = 4$

$\therefore x = -\frac{9}{4}, 4$

[Remark : (1) Exprensing $6\frac{6}{7}$ as $7 - \frac{1}{7}$ we could have written the equation as $\frac{x-3}{x+3} - \frac{x+3}{x-3} = -7 + \frac{1}{7}$ and from the reciprocal relation we will get

$\frac{x-3}{x+3} = -7$ or, $\frac{x-3}{x+3} = \frac{1}{7}$

from which we could have solved the equation.

(2) To express any fraction as sum or difference of the fractions such that one is reciprocal of the other, resolve the denominator in two factors, sum or difference of whose squares are the numerator of the fraction respectively]

Exercise 23

Solve :

1. $25x^2 = 100$
2. $(x + 8)(x - 8) = 36$
3. $4x^2 + 5 = 105$
4. $3y^2 - 20 = 160 - 2y^2$
5. $(2x + 1)^2 + (x + 1)^2 = 6x + 47$

6. $3x - \frac{2}{x} = \frac{x}{3}$

7. $\frac{2x+1}{x+1} = \frac{x+8}{x+4}$

8. $\frac{x}{3} + \frac{3}{x} = \frac{15}{x}$

9. $\frac{x+3}{x-3} + \frac{x-3}{x+3} = 2\frac{1}{2}$

10. $(x-7)(x-9) = 195$

11. $\frac{x}{a} + \frac{a}{x} = \frac{x}{b} + \frac{b}{x}$

12. $\frac{ax+b}{a+bx} = \frac{cx+d}{c+dx}$

13. $6x^2 + 11x - 10 = 0$

14. $42x^2 - 41x - 20 = 0$

15. $5x^2 - 4x - 12 = 0$

16. $(17x-8)(x-2) = 555$

17. $\frac{x-2}{x+2} + 6\left(\frac{x-2}{x-6}\right) = 1$

18. $\frac{x}{x+1} + \frac{x+1}{x} = 2\frac{1}{12}$

19. $\frac{12x+17}{3x+1} - \frac{2x+15}{x+7} = 3\frac{1}{5}$

20. $\frac{x+1}{2} + \frac{2}{x+1} = \frac{x+1}{3} + \frac{3}{x+1} - \frac{5}{6}$

21. $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$

22. $\frac{x+3}{x-3} + 6\left(\frac{x-3}{x+3}\right) = 5$

23. $x^2 - x - 650 = 0$

24. $(2x+1) + \frac{3}{2x+1} = 4$

25. $\left(\frac{x+a}{x-a}\right)^2 - 5\left(\frac{x+a}{x-a}\right) + 6 = 0$

26. $\frac{1}{x} - \frac{1}{x+b} = \frac{1}{a} - \frac{1}{a+b}$

27. $\frac{(x+1)^3 - (x-1)^3}{(x+1)^2 - (x-1)^2} = 2$

28. $\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} + \frac{1}{(x-3)(x-4)} = \frac{1}{6}$

29. $\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}$

QUADRATIC EQUATION

Problems on Quadratic Equations

You know how to form a quadratic equation. In the following examples you will find how different problems can be worked out by solving quadratic equations.

Example 1 : The length of a rectangle is greater than its breadth by 3 m. If its area be 10 sq. m, find the perimeter.

Suppose, the breadth of the rectangle = x m.

$\therefore$ length of the rectangle = (x + 3) m.

so, area = (x + 3)x sq. m.

Hence, by the condition of the problem,

$$(x+3)x = 10$$
$$\text{or, } x^2 + 3x = 10$$
$$\text{or, } x^2 + 3x - 10 = 0$$
$$\text{or, } (x+5)(x-2) = 0$$
$$\text{so, } x = -5, 2$$

But $x = -5$ is not acceptable, since breadth cannot be negative.

Therefore. $x = 2$

Hence, breadth = 2 m.

and length = 5 m.

$\therefore$ Perimeter = 2(2 + 5) m

= 14 m.

$x = -5$ does not satisfy the conditions of the problem; such a root is called an extraneous root. In solving a problem, each root of the quadratic equation is to be verified whether it satisfies the conditions of the given problem. An extraneous root is to be rejected.

Example 2 : Find a positive number, which is less than its square by 30.

Let the number be x

∴ by the condition, $x^2 - x = 30$

or, $x^2 - x - 30 = 0$

or, $(x - 6)(x + 5) = 0$

tf $x = 6, -5$

As the number is positive, $x = -5$ is not acceptable. Thus the required number is 6.

Example 3 : The hypotenuse of a right-angled triangle is 13 cm; if the difference of its other two sides be 7 cm, find the lengths of other two sides.

Let the length of the smallest side be x cm.

∴ the length of another side is $(x + 7)$ cm.

By the property of a right-angled tringle,

$(x + 7)^2 + x^2 = 13^2$

or, $x^2 + 14x + 49 + x^2 = 169$

or, $2x^2 + 14x - 120 = 0$

or, $x^2 + 7x - 60 = 0$

or, $x^2 + 12x - 5x - 60 = 0$

or, $(x - 5)(x + 12) = 0$

So, $x = 5, -12$.

As length cannot be negative, the length of the smallest side is 5 cm. The length of the other side is $(5 + 7)$ cm or 12 cm.

Example 4 : The product of two consecutive even numbers is 528; find the numbers.

Let the numbers be $2x$ and $2x+2$. By the condition of the problem.

$2x(2x + 2) = 528$

or, $2x \times 2(x + 1) = 528$

or, $x(x + 1) = 132$

or, $x^2 + x - 132 = 0$

or, $(x + 12)(x - 11) = 0$

Therefore, $x = -12$ or 11.

Thus, the numbers are -24 and -22 or, 22 and 24.

Remark : As the numbers are even, they are taken as $2x$ and $2x + 2$. However, they could have also be taken as x and $x + 2$. With x and $x + 2$ as two numbers solve the problem.

Example 5 : A cyclist after travelling 84 km. finds that had he travelled 5 km an hour more, then he would have taken 5 hours less. At what speed in km/hour has the cyclist travlled ?

Suppose, the cyclist has travelled with a speed of x km/hour.

$\therefore$ by the condition, $\frac{84}{x} - \frac{84}{x+5} = 5$

or, $\frac{84(x+5-x)}{x^2+5} = 5$

or, $\frac{84.5}{x^2+5} = 5$

or, $\frac{84}{x^2+5} = 1$

or, $x^2 + 5x = 84$

or, $x^2 + 5x - 84 = 0$

or, $(x + 12)(x - 7) = 0$

$\therefore x = -12, 7.$

But, $x \neq -12$, as rate of travelling cannot be negative . So $x = 7$.

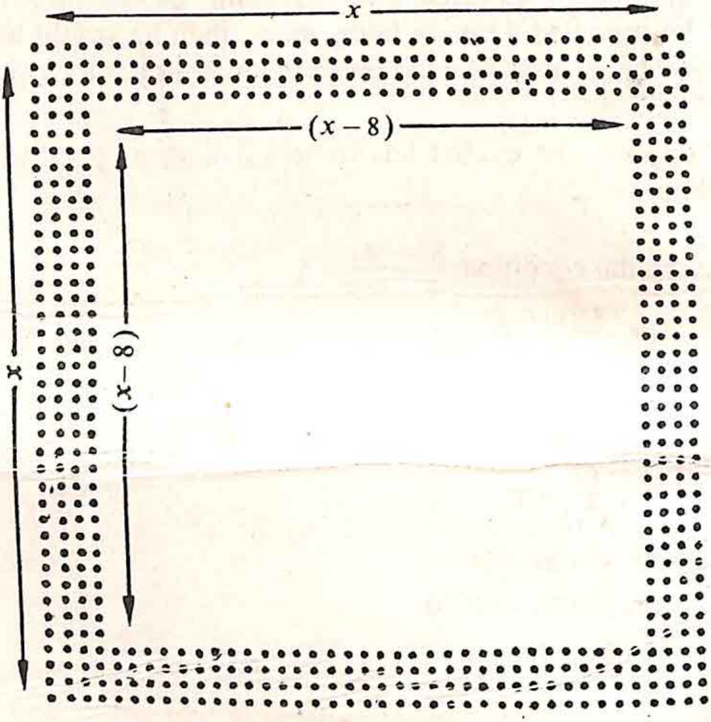
Thus, the cyclist has travelled with a speed of 7 km/hour.

Example 6 : A regiment of soldiers when arranged in a solid square has 16 soldiers less in the front row than when arranged in a hollow square 4 deep; find the number of soldiers in the regiment.

Suppose, the number of soldiers in the front row of the hollow square be x.

Therefore, the number of soldiers is

$$x^2 - (x - 2 \times 4)^2$$
$$= x^2 - (x - 8)^2.$$



Again, when arranged in a solid square, the number of soldiers is $(x - 16)^2$.

Thus, $x^2 - (x - 8)^2 = (x - 16)^2$

or, $x^2 - x^2 + 16x - 64 = x^2 - 32x + 256$

or, $-x^2 + 48x - 320 = 0$

or, $x^2 - 48x + 320 = 0$

or, $x^2 - 40x - 8x + 320 = 0$

or, $(x - 40)(x - 8) = 0$

$\therefore$ $x = 40, 8$

But $x \neq 8$, since the number of soldiers in the front row of a hollow-square 4 deep, must be greater than 8. $\therefore x = 40$. Hence the number of soldiers in the regiment

$$= (x-16)^2$$
$$= (40-16)^2$$
$$= 24^2$$
$$= 576.$$

Exercise 24

1. The difference of two positive integers is 3 and the sum of their squares is 117; find the numbers.

2. The length of a rectangular field is greater than its breadth by 36 m. If the area of the field is 460 sq. m, find its length.

3. The base of a triangle is greater than twice its altitude by 18 m.; if the area of the triangle is 360 sq. m, find the altitude.

4. Five times of a positive integer is less than its square by 3; find the integer.

5. A rectangular garden, 40 m long and 30 m wide is surrounded by a path of uniform width, in its interior. If the area of a garden excluding the path be 375 sq. m, find the width of the path.

6. Two places are 200 km apart; a jeep takes 2 hours less time in travelling from one place to another than by motor car. If the speed of the jeep be greater than that of the motor car by 5 km/hour, find the speed of the motor car.

7. The area of a rectangular field is 2,000 sq. m and its perimeter is 180 m; find the length and breadth of the field.

8. The speed of a goods train is less than that of an express train by 15 km/hour; if the goods train takes 3 hours 45 minutes time more than the expres train in going through 175 km, find the speed of the express train.

9. A cistern is filled by two pipes in $11\frac{1}{2}$ minutes. If the pipes are opened separately, one pipe takes 5 minutes more than the other in filling the cistern. Find how of much time will be taken by the pipes to fill the cistern separately.

10. A boatman after rowing to a place of, distance 36 km, came back to his original place. He took 1 hour more in returning than going to the place; if the speed of the current be 3 km/hour, find the speed of the boat in still water.

11. *A* and *B* complete a piece of work in 4 days. Had they worked separately, *B* would have taken 6 days more than *A*. In how many days can *A* do it alone ?

12. If the price of pins be diminished by 2 paise per dozen, 6 more pins are available in 42 paise. Find the present price of pins per dozen.

CHAPTER 9

INEQUATION

Inequality : Whether a number is equal to or greater than or less than another number can be said by their diference. Thus if,

$(a-b)=0$, then $a = b$

$(a-b)$ is positive, then $a > b$

$(a-b)$ is negative, then $a < b$

Since a positive number is greater than zero and a negative number is less than zero, we can write

when $a > b$, then $a-b > 0$

when $a < b$, then $a-b < 0$

Thus we can write, in general;

If, $a-b=0$, then $a=b$

$a-b>0$, then $a>b$

$a-b<0$, then $a<b$

Now, $a=b$ is an equality, where as $a>b$ or $a<b$ are inequalities. Besides the symbols '>' or '<', inequality is also expressed by some other symbols, such as,

$a \neq b$ (a is not equal to b)

$a \geq b$ (a is greater than or equal to b, i.e. the minimum value of a is b.)

$a \leq b$ (a is less than or equal to b. i.e. the maximum value of a is b.)

$a > b$ (a is not greater than b)

$a < b$ (a is not less than b.)

In practice, inequality has more applications than equality. Some properties of the inequality are discussed below.

(1) If $a > b$, then $-a < -b$

Proof : Since $a > b$, $a-b > 0$.

Now, $-a-(-b) = -a+b = -(a-b)$ which is negative ?

$\therefore -a < -b$.

Similarly, it can be shown that if $a < b, -a > -b$.

(2) If $a > b$, then $a + c > b + c$.

Proof : $(a + c) - (b + c) = a - b > 0$.

$\therefore a + c > b + c$

Similarly, if $a < b, a + c < b + c$

(3) If $a > b$ then $a - c > b - c$

Proof : $(a - c) - (b - c) = a - b > 0$

$\therefore a - c > b - c$

Similarly, if $a < b$, then $a - c > b - c$.

(4) If $a > b$, then $ac > bc; (c > 0)$.

Proof : $ac - bc = c(a - b) > 0$

Similarly, when $a < b, ac < bc; (c > 0)$

(5) If $a > b$, then $\frac{a}{c} > \frac{b}{c}; (c > 0)$

Proof : $\frac{a}{c} - \frac{b}{c} = \frac{1}{c}(a - b) > 0$

$\therefore \frac{a}{c} > \frac{b}{c}$.

Similarly, when $a < b, \frac{a}{c} < \frac{b}{c}, (c > 0)$

Examine yourself what happens to the properties (4) and (5) , when $c < 0$.

(6) If $a > b$ and $b > c$, then $a > c$

Proof : $a - c = (a - b) + (b - c) > 0$

$\therefore a > c$

Similarly, when $a > b$ and $b < c$, $a < c$.

Remark ; The inequalities $a > b$ and $b > c$ can be written together as $a > b > c$.

Similarly, the inequalities $a < b$ and $b < c$ can be written together as $a < b < c$

Inequations : You have seen how to form an equation and to solve it. In an equation the involving known and unknown quantities x, y etc. are connected with known quantities through the sign of equality, '='. Thus,

$x = 5$ ...... (1)

$2x + 3y = 5$ ....(2)

$x^2 + x - 2 = 0$ .... (3)

are three different equations.

But in all cases, it is not possible to express the relationship between expressions involving known and unknown quantities using the sign of equality. For example, suppose, more then 500 students read in your school, then the number of students cannot be expressed by using the sign of equality, since the exact number of students reading in the school is not stated. But if we take that there are x students in the school, we can write, $x > 500$.

Similarly, if a pen costs more than Rs. 2 and an exercise book costs less than Rs. 3 we can write,

$x > 2; x =$ cost of one pen in rupees

$y < 3; y =$ cost of one exercise book in rupees

In each of the above cases the relation between the known (number) and the variable has been expressed by means of the sign of inequality. Such a relationship between the variable denoted by algebraic symbols, and the known, expressed by the signs of inequality $<, >, \geq, \leq, \neq$, is called an inequation.

So, each of $x > 500$, $x > 2$, $y < 3$ stated above, is an inequation.

Further examples of inequation :

(1) $2x + 3y \leq 12$

(2) $x + y > 5$

(3) $5x + 3y \geq 30$

$3x \leq y - 2$ etc.

Basic difference between equation and inequation :

A linear equation has one root and a quadratic equation has two roots. You will learn later on that the number of roots of an equation is equal to the highest degree of the unknown (quantity) in the equation. So, the number of roots of an equation is finite. But this does not apply to the case of an inequation. Thus, the inequation $x < 5$ is satisfied for any value of x less than 5, and the number of numbers less than 5 is infinite. Similarly, the inequation $2x - 3 \geq 5$ holds for an infinite number of values of x. Hence, an inequation is satisfied by an infinite values of the unknown quantity. The set of values for which an inequation is satisfied is called the solution set of the inequation.

Remark ; An inequation with a given condition may have a finite number of solutions.

For example, for x a positive integer, $x \leq 5$ has the solution, 1, 2, 3, 4, 5. Here we can denote the solution set of the inequation, $x \leq 5$ by $\{1, 2, 3, 4, 5\}$

Simple inequation ; If an inequation contains an unknown quantity of power one, it is called, a simple inequation. $2x + 3 > 7$, $5x - 2 \geq 18$, $3x - 5 \leq 10$ etc. are simple inequations.

By applying the properties of inequality discussed earlier, we can easily solve simple inequations. This can be seen in the following examples.

Example 1. Solve : $4x - 8 \leq 12$

$$4x - 8 \leq 12$$

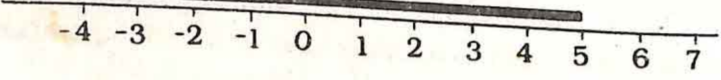
$$\text{or, } 4x - 8 + 8 \leq 12 + 8$$

$$\text{or, } 4x \leq 20$$

$$\text{or, } \frac{4x}{4} \leq \frac{20}{4}$$

$$\text{or, } x \leq 5.$$

$\therefore$ the required solution is $x \leq 5$



This is the reqired solution. Thus the given inequation is satisfied by 5 or any number less than 5. We can show these values on the number line. Fig. 20

Example 2. Solve : $2(x-4) \geq 3x-5$

$2(x-4) \geq 3x-5$

or, $2x-8 \geq 3x-5$

or, $2x-3x \geq 8-5$

or, $-x \geq 3$

or, $x \leq -3$.

$\therefore$ the required solution is $x \leq -3$

This is the required solution. These values are shown on the number line.(Fig 21)

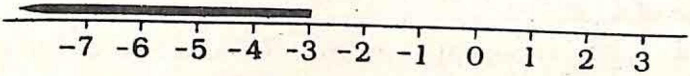


Fig. no. 21

Remark : As a result of dividing both sides of $-x > 3$ by -1, '≥' sign is converted into '≤' sign.

Example 3. Solve.: $-5 \leq 2x-7 \leq 1$

Here, two inequations are given :

$-5 \leq 2x-7$ ...... (1)

and $2x-7 \leq 1$ ....(2)

From inequation (1) we get $-5+7 \leq 2x$

or, $2 \leq 2x$

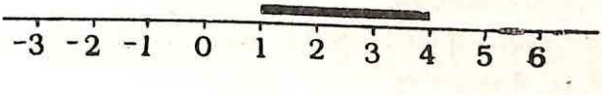
or, $x \geq 1$

From inequation (2) we get $2x-7 \leq 1$

or, $2x \leq 8$

or, $x \leq 4$

The required solutions are $x \geq 1$, $x \leq 4$. that is. $1 \leq x \leq 4$. We can show these values on the number line (Fig. 22)



Exercise 25

1. Solve the following inequations and show the solutions on the number line.

(i) $5 - 2x > 1$,

(ii) $\frac{3}{x} - \frac{5}{4} > 0$

(iii) $6x + 19 < 3x - 1$,

(iv) $\frac{1}{2}x + \frac{2}{3} \geq \frac{5}{4}$

(v) $\frac{2}{3x} + \frac{2}{3} \leq \frac{5}{2} + \frac{1}{2x}, (x > 0)$

(vi) $\frac{7}{2x} - \frac{5}{7} > \frac{5}{3x} - \frac{2}{3}, (x>0)$

(vii) $\frac{7x+9}{5} \geq \frac{5x+2}{4}$

2. From the inequation, $2x - 5 \geq 16$, find the minimum value of x.

3. From the inequation, $5x + 7 \leq 3x + 9$, find the maximum value of x.

4. If x is a positive integer, find the solution-set of the inequation $30 - 4(2x + 1) < 30$.

5. If x is a negative integer, find the solution set of the inequation $12 + 6x > 0$.

6. If x and y are two whole numbers, where $x>0$ and $y>1$, then write the solution set of the inequation $2x+37\leq10$ in the following table

x		
y		

7. If x,y be two positive integers and $x < 2$, $y < 3$, then write the solution-set of the inequation $3x + 4y \geq 6$ in the following table :

x		
y		

8. Find the solution-set of the inequation $4x + 2y < 11$, where x and y are positive integers.

9. If x,y be positive integers or zero, find the solution-set of the inequation : $9 > 4x + 3y$.

* * *

Graphs of inequations (Two dimensional)

You know that the co-ordinates of a point lying on the graph of an equation satisfy it, whereas those of any outside point do not. The graph of the equation $x = 5$ is a straight line. The x co-ordinate of any point on this graph is 5. Let us now see how the graph of the inequation $x \geq 5$ can be drawn.

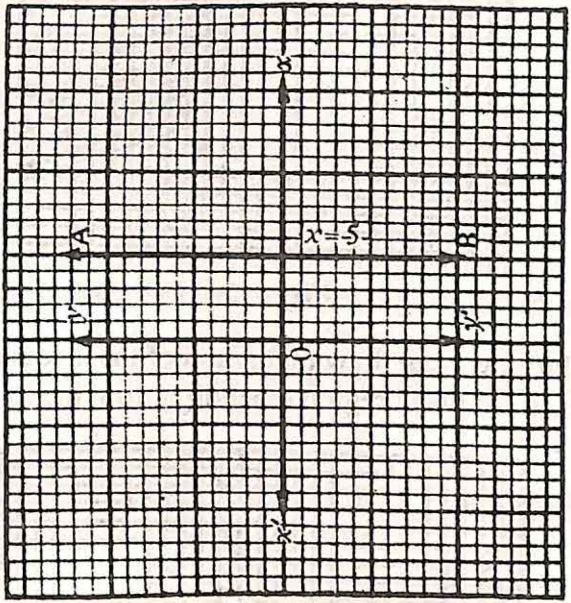


Fig. 23

In fig. 23 the line AB is the graph of the equation $x = 5$.

The x co-ordinates of any point lying on the plane to the right of the line AB is greater than 5, whereas for every point on AB, it is 5. Thus for all points of the plane lying on AB and to the right of AB, the inequation $x \geq 5$ is satisfied.

That is, the plane region extending from the line AB to the right of it indicates the solution-set of the inequation $x \geq 5$.

The solution region is shown in Fig. 24 by shaded line segments.

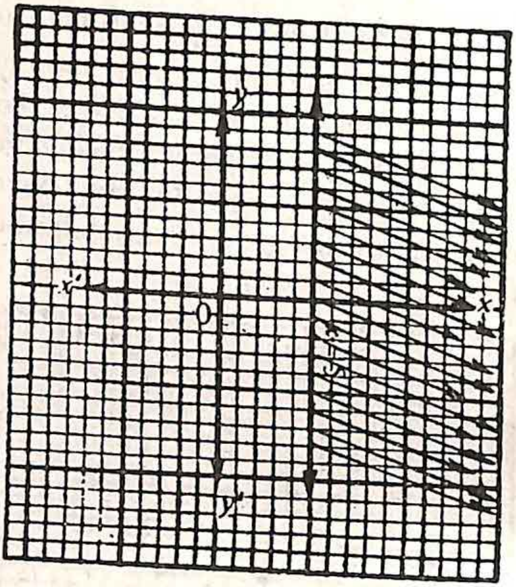


Fig. 24

Similarly, we can draw that graph of the inequation $x \leq 5$. In this case the plane region extending from the line $x = 5$ to

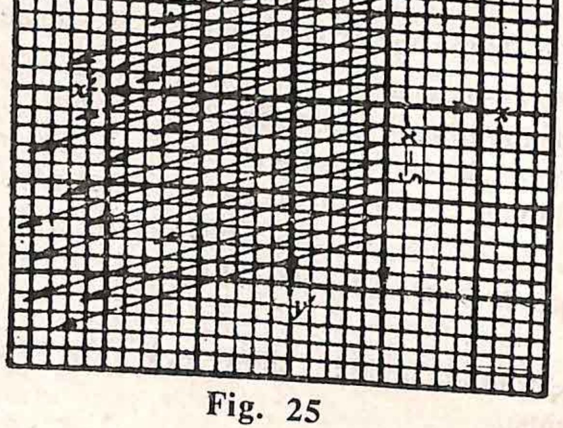


Fig. 25

the left of it indicates the solution-set of the inequation $x \leq 5$. (Fig. 25)

In this case, the solution region is shown in Fig. 11 by shaded line segments (Page).

From the above discussion we can easily understand the solution-set of the inequation $x>5$ is denoted by the set of points lying to the right of the line $x = 5$ excepting those which are situated on the line $x = 5$.

Similarly, the solution-set of the inequation $x < 5$ is denoted by the set of points lying to the left of the line $x = 5$ excepting those which are on that line.

Thus we see that the line $x = 5$ divides the plane into two regions. One of these regions indicates the solution-set of the inequation $x >5$ and the other, the solution-set of $x<5$, whereas the points on the line give the solution-set of the equation $x = 5$. This is true in all cases.

It is understood from above discussion that, to draw the graph of an inequation, we are draw the graph of the corresponding equation and then to verify which side of the graph satisfies the inequation. This region indicates the solution-set of the given inequation.

Remark : The graph of a linear equation is a straight line, but the graph of a linear inequation is a plane region.

Example 1 : Draw the graph of $y \leq 5$. The equation corresponding to this inequation is $y = 5$.

The graph of $y = 5$ is drawn, (Fig. 26). The set of points lying on the graph and below it satisfies the inequation $y \leq 5$. So the region extending from $y = 5$ and below it indicates the solution-set of the inequation.

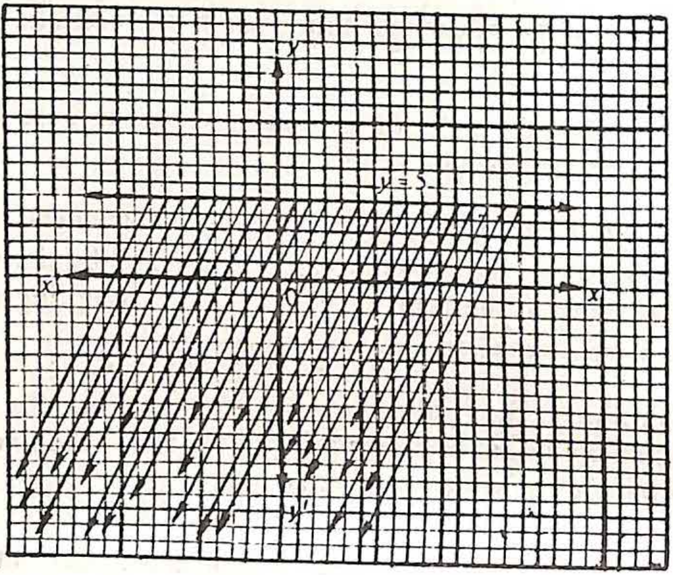


Fig. 26

(The shaded region gives the solution-set of $y \leq 5$).

We can find the solution-region in another way. Let us take any point say (10, 10) outside the line $y = 5$. This point does not satisfy the inequation $y \leq 5$. So, the side of the line $y = 5$, other than in which the point (10, 10) is situated indicates the solution-set of the given inequations.

Graphs of the inequations of the form

(i) $ax + by + c \geq 0$

(ii) $ax + by + c \leq 0$

(iii) $ax + by + c < 0$

(iv) $ax + by + c = \leq 0$

The equation corresponding to these inequations

$ax+by+c = 0$. You know that the graph of an equation of the form $ax+by+c = 0$ is a straight line. This straight line divides the plane into two regions. Of these two regions, one satisfies the inequation $ax+by+c > 0$ and the other,

$ax+by+c<0$. Which side satisfies the solution-set of which inequation can be understood from the following example.

Example 2 : Draw the graph of the inequation $2x+3y \le 12$. The equation corresponding to the given inequation is

$$2x + 3y = 12.$$

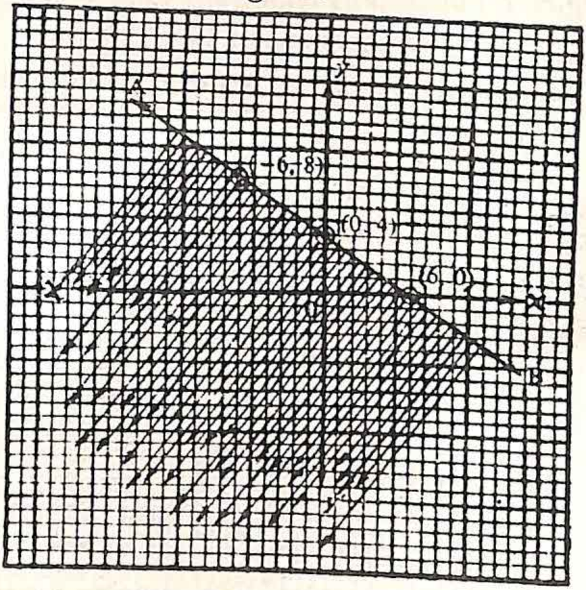
or, $3y = 12 - 2x$

or, $y = \frac{12-2x}{3}$

x	6	0	-6
y	0	4	8

By plotting the points (6, 0), (0, 4) and (–6, 8), the graph AB of $2x + 3y = 12$ is obtained. The line AB divides the plane into two regions, one of those two regions will satisfy the inequation $2x + 3y < 12$. You can proceed in the following way to determine which region will indicate the solution-set of the given inequation.

Fig. 27



Take any point that is not situated on the line AB. Put the co-ordinates of the point for x and y in the inequation and see whether they satisfy the inequation. If the inequation is satisfied, then the region on the side of AB, where the point is situated, indicates the solution-set of the inequation, otherwise, the region on the other side of AB will indicate the solution-set.

Let us examine with the origin. Here, the origin O, is a point outside AB. Putting $x = 0, y = 0$ in the inequation $2x+3y<12$, we get $2.0 + 3.0 < 12$. i.e., $0 < 12$. Thus, the co-ordinates of the origin satisfy the inequation. So, the region on that side of AB which contains the origin indicates the solution-set of the given inequation. This region is marked by drawing line segments. You may verify whether the same region is obtained by taking any other point.

Example 3 : Draw the graph of $5x - 2y \geq 6$

The equation corresponding to the given inequation is

$5x - 2y = 6$ ... (i)

or, $2y = 5x - 6$

or, $y = \frac{5x-6}{2}$

x	0	2	4
y	-3	2	7

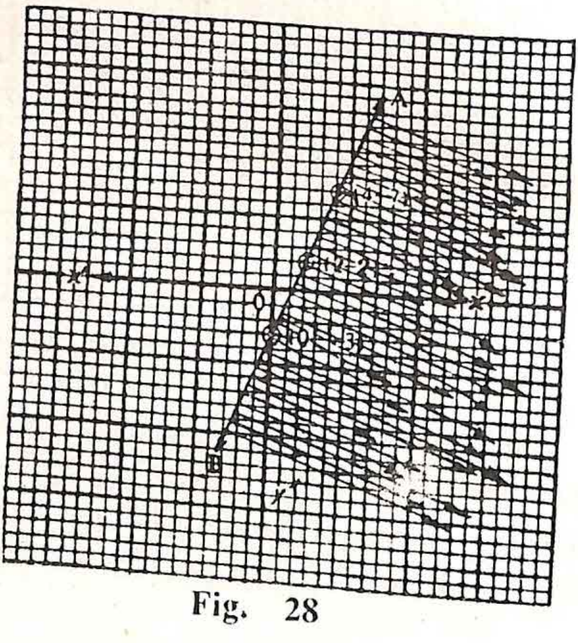


Fig. 28

The graph of the equation (i) is drawn. Here, origin lies outside the graph. Putting $x=0, y=0$ in the inequation $5x-2y>6$ it is found that the inequation is not satisfied. So the side of the graph of the equation in which the origin does not lie indicates the solution set. It is marked by drawing line segments (fig. 28).

Remark : The side in which the origin lies is called, in brief, origin side and other side as non-origin side.

Simultaneous inequations ; You are acquainted with simultaneous equations and you know that the two simultaneous equations give together difinite solution, which is the common solution of each of the equations. Similarly, if more than one inequation has a common solution-set, then the inequations are called simultaneous inequations.

On drawing graphs of simultaneous inequations it is found that they are satisfied by all the points in a common region. This common region indicates the solution-set of the simula-taneous inequations.

How the solution-set of simultaneous inequations is indicated by drawing graphs, is illustrated in the following examples.

Example 4 : Solve graphically

$$x \geq 5, y \geq 6$$

The corresponding equations are

$$x = 5 \text{ and } y = 6.$$

Taking the same axes, the graphs of the equations $x = 5$ and $y = 6$ are drawn. The solution-region of $x \geq 5$ and $y \geq 6$ are marked by drawing line segments. The common portion

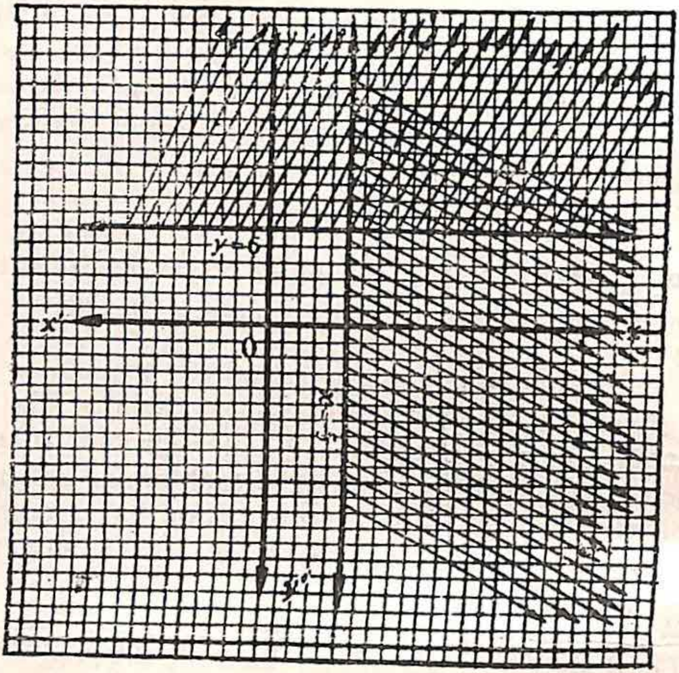


Fig. 29

of the two solution-regions indicates the solution-set of the two given simultaneous inequations.

Examples 5 : Solve graphically

$$3x - 2y \geq 12 \text{ and } 2x - y \leq 6$$

The corresponding equations are

$$3x - 2y = 12 \qquad \text{.... (1)}$$

and $2x - y = 6$ .... (2)

From equation (1)we get

$$2y = 3x - 12$$

$$\text{or, } y = \frac{3x-12}{2}$$

x	0	4	–4
y	–6	0	–12

From equation (2), we get $y = 2x - 6$,

x	0	3	5
y	-6	0	4

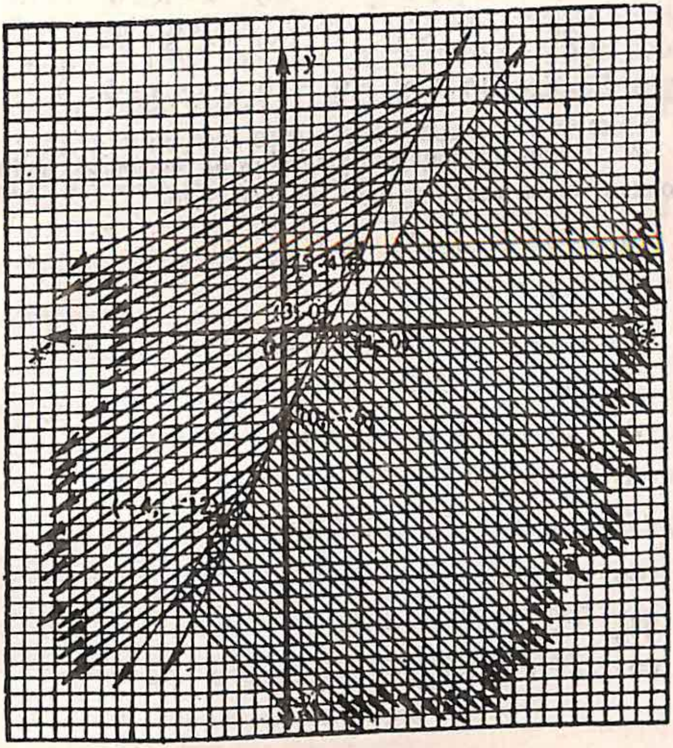


Fig . 30

The graphs of the equations (1) and (2) are drawn (fig. 30).

The solution-regions of the given inequations are marked by drawing line segments. The intersection of the two regions indicate the solution-set of the given simultaneous inequations.

Exercise 25

1. Draw the graps of :

   (i) $x \geq 4, y \leq 1$ (ii) $x - 5 \leq 0$

   (iii) $-5 \leq x \leq 5$ (iv) $-3 \leq y \leq 4$

   (v) $2x + 3y + 5 \geq 0$ (vi) $x + y - 5 \leq 0$

2. Draw the graphs of the following inequations and mark the solution-region satisfying them simultaneously.

   (i) $-3 \leq x \leq 5, -5 \leq y \leq 5$

   (ii) $y \geq x, y + x \leq 2$

   (iii) $2x + 5y \leq 9, x - 2y + 2 \geq 0$

   (iv) $3x - 2y \geq 12, 2x - y \leq -6$

   (v) $x + y \leq 10, x \geq 1, y \geq 0$

   (vi) $x + 4y \leq 12, x \geq 0, y \geq 0$

   (vii) $x \leq 1, y \leq 2, x - 4y \leq 12$

   (viii) $x \geq 2, y \geq 2, x + y \leq 10$

   (ix) $y \geq 4, x \geq 2, 2x + y \leq 10$

   (x) $x + y - 5 \leq 0, 2x - 3y - 6 \geq 0, x - 2 \geq 0$

Chapter 10

Surds

You know square of 2 is 4 (in mathematical language, $2^2 = 4$), so the square root of 4 is 2. Again, since $(-2)^2 = 4$, another square root of 4 is -2. From the concept of square root you can have the idea of cube root, fourth root etc. Similarly, since cube of 2 is 8 (in mathematical language $2^3=8$), 2 is a cube root of 9. Again, since $3^3 = 27$ and $4^3 = 64$, 3 and 4 are cube roots of 27 and 64 respectively.

Besides, 2,3,4 there are other cube roots of 8, 27, 64 respectively. You will learn more about this later on.

Similarly,

Since, $5^3 = 125$, 5 is a cube root of 125.

$3^4 = 81$, 3 ia a fourth root of 81.

$2^5 = 32$, 2 is a fifth root of 32.

You know the symbol of square root. Square root of 2 is $\sqrt{2}$; square root of 5 is $\sqrt{5}$ etc. Similarly, cube root is denoted by $\sqrt[3]{\ }$ fourth root by $\sqrt[4]{\ }$ and fifth root by $\sqrt[5]{\ }$.

The roots of 125, 81 and 32 mentioned above are completely known. But it is not so in all cases.

$\sqrt{2} = 1.414$ ...., $\sqrt{3} = 1.732$ ....

$\sqrt{5} = 2.236$........

You have seen earlier, the square roots of 2, 3, 5 are not completely known.

Similarly, $\sqrt[3]{7}$, $\sqrt[4]{20}$, $\sqrt[5]{25}$ etc. cannot be completely determined. The roots of numbers that cannot be completely determined are called Surs. Thus, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt[3]{7}$, $\sqrt[4]{20}$ are surds. But $\sqrt{4}$, $\sqrt[3]{8}$, $\sqrt[3]{27}$ are not surds, since their values are completely determined.

Order of Surds : Being related to square of second power, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ etc. are called quadratic surds.

Similarly, $\sqrt[3]{2}$, $\sqrt[3]{7}$ are called surds of third order or cubic surds, $\sqrt[4]{5}$, $\sqrt[4]{15}$ aresurds of fourth order or biquadratic surds etc. We shall discuss only quadratic surds here.

Rational and irrational numbers : If a number can be expressed as the ratio of two intergers it is called a rational number. Thus, $-3, 1, 10, \frac{2}{3}, \frac{-2}{3}$ are rational numbers.

The numbers which cannot be expressed as the ratio of two intergers are called irrational numbers. Thus, $\sqrt{2} = 1{\cdot}414$ cannot be expressed as the ratio of two integers. So, $\sqrt{2}$ is an irrational number. Similarly, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{11}$ are irrational numbers. No surd can be expressed as the ratio of two integers and so every surd is an irrational number.

Pure and mixed Surds :

Look at thr following surds.

$$\begin{array}{lllll}
\sqrt{2} & = & \sqrt{4\times2} & = & 2\sqrt{2} \\
\sqrt{18} & = & \sqrt{9\times2} & = & 3\sqrt{2} \\
\sqrt{48} & = & \sqrt{16\times3} & = & 4\sqrt{3} \\
\sqrt{125} & = & \sqrt{25\times5} & = & 5\sqrt{5}
\end{array}$$

$\sqrt{8}$, $\sqrt{18}$, $\sqrt{48}$, $\sqrt{125}$ are found to have a product of a rational and an irrational part. Such surds are called mixed surds. But, from the surds $\sqrt{5}$, $\sqrt{11}$, $\sqrt{17}$ we do not have any rational part. Such surds are called *pure surds.*

Similar and Dissimilar Surds : Two surds (on simplification, if necessary) whose irrational parts are same are called like or similar surds. Thus numbers in each pair of the following surds $\sqrt{2}$ and $3\sqrt{2}$, $2\sqrt{7}$ and $5\sqrt{7}$, $3\sqrt{11}$ and $7\sqrt{11}$ are similar to each other.

Again $\sqrt{12}$ and $\sqrt{27}$ are also two similar surds, since,

$$\sqrt{12} = \sqrt{4\times3} = 2\sqrt{3} \text{ and } \sqrt{27} = \sqrt{9\times3} = 3\sqrt{3}$$

If the irrational parts of two surds are not same, they are called unlike or dissimilar surds. Thus numbers in each pair of the following surds are dissimilar to each other : $\sqrt{3}$ and $\sqrt{5}$, $2\sqrt{3}$ and $2\sqrt{7}$, $\sqrt{48}$ and $\sqrt{50}$.

Simple and Compound Surds ; When a surd consists of one term such as $\sqrt{2}$, $\sqrt{3}$, $2\sqrt{7}$ it is called a simple surd. The algebraic sum of two or more simple surds or that of a rational numbers and a simple surds is called a compound surd. Thus, $1+\sqrt{5}$, $\sqrt{2}+\sqrt{3}$, $2\sqrt{2} + 3\sqrt{5}+\sqrt{6}$ are compound surds. The alebraic sum of two surds or that of a rational number and a surd is called a binomial surd. Thus, $\sqrt{2} + \sqrt{5}$, $3 + \sqrt{5}$, $7 - \sqrt{3}$ are binomial surds.

Addition and subtraction of surds :

You know, $a + 2a = 3a$

$$3x + 6x = 9x$$

$$5b - 2b = 3b \text{ etc.}$$

Likewise, we can find the sum and difference of two similar surds. Thus,

$$\sqrt{2} + 2\sqrt{2} = (1 + 2)\sqrt{2} = 3\sqrt{2}$$

$$2\sqrt{3} + 5\sqrt{3} = (2 + 5)\sqrt{3} = 7\sqrt{3}$$

$$5\sqrt{2} - 2\sqrt{2} = (5 - 2)\sqrt{2} = 3\sqrt{2}$$

Thus in the sum or difference of two similar surds, the irrational part is the same as that of the given surds and the rational part is the sum or difference of the rational parts of the given surds.

You know, sum of a and $b = a + b$

Sum or 2x and $3y = 2x + 3y$

When a is subtracted from x, the difference $= x - a$.

Similarly, the sum of two dissimilar surds $\sqrt{2}$ and $\sqrt{5}$ is $\sqrt{2} + \sqrt{5}$. The sum of $3\sqrt{2}$ and $3\sqrt{3}$ is $3\sqrt{2} + 3\sqrt{3}$. When $\sqrt{3}$ is subtracted from $\sqrt{5}$, the difference is $\sqrt{5} - \sqrt{3}$.

Product of Surds :

You know, a number, which is obtained by taking square root of another number, when squared, gives the original number.

Thus, $\sqrt{9} = 3$, again $3^2 = 9$

$\sqrt{16} = 4$, again $4^2 = 16$ etc.

Similarly, we can write,

Square root of 2 is $\sqrt{2}$, so, $(\sqrt{2})^2 = 2$

Square root of 3 is $\sqrt{3}$, so, $(\sqrt{3})^2 = 3$

Square root of 11 is $\sqrt{11}$, so, $(\sqrt{11})^2 = 11$

Thus you see,

$\sqrt{2} \times \sqrt{2} = (\sqrt{2})^2 = 2$

$\sqrt{3} \times \sqrt{3} = (\sqrt{3})^2 = 3$

$\sqrt{11} \times \sqrt{11} = (\sqrt{11})^2 = 11$

Again, $\sqrt{6} = \sqrt{3 \times 2}$

But $6 = 3 \times 2 = (\sqrt{3})^2 \times (\sqrt{2})^2 = (\sqrt{3} \times \sqrt{2})^2$

$\therefore \sqrt{6} = \sqrt{3} \times \sqrt{2}$

Thus, $\sqrt{3} \times \sqrt{2} = \sqrt{3 \times 2} = \sqrt{6}$

Similarly, $\sqrt{7} \times \sqrt{3} = \sqrt{7 \times 3} = \sqrt{21}$

$\sqrt{8} \times \sqrt{2} = \sqrt{16} = 4$

In general, $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

Again, $2 \times \sqrt{3} = \sqrt{4} \times \sqrt{3} = \sqrt{12} = 2\sqrt{3}$

$3\sqrt{7} \times 2\sqrt{3} = 3 \times \sqrt{7} \times 2 \times \sqrt{3}$

$= 6 \times \sqrt{21} = 6\sqrt{21}$

Thus, to get the product of two surds, the rational and the irrational parts of the surds are multiplied separately. The product of the number of irrational parts is written within the radical sign and this may be reduced to simpler forms, if necessary.

For example, $2\sqrt{12} \times 3\sqrt{8} = 6\sqrt{96}$

$$= 6\sqrt{16\times6} = 6 \times 4\sqrt{6}$$

$$= 24\sqrt{6}$$

In general, $a\sqrt{b} \times c\sqrt{d} = ac\sqrt{bd}$

You know how to find the product of two algebraic expressions. In a similar way we can find the product of two compound surds. for example,

$$(2+\sqrt{3})(4+\sqrt{3}) = 8 + 2\sqrt{3} + 4\sqrt{3} + (\sqrt{3})^2$$

$$= 8 + 6\sqrt{3} + 3$$

$$= 11 + 6\sqrt{3}$$

$$(3+\sqrt{5})(2+\sqrt{3}) = 6 + 3\sqrt{3} + 2\sqrt{5} + \sqrt{15}$$

$$(2+\sqrt{3}+\sqrt{5})(3-\sqrt{5}) = 6 - 2\sqrt{5} + 3\sqrt{3} - \sqrt{15} + 3\sqrt{5} - 5$$

$$= 1 + \sqrt{5} + 3\sqrt{3} - \sqrt{15}$$

$$(3+\sqrt{7})(3+\sqrt{7}) = (3+\sqrt{7})^2$$

$$= 9 + 6\sqrt{7} + (\sqrt{7})^2$$

$$= 9 + 6\sqrt{7} + 7$$

$$= 16 + 6\sqrt{7}$$

Conjugate or Complementary Surds :

Two binomial surds consisting of the same terms but connected by opposite signs are calles conjuugate or complementary to each other.

Thus, $2+\sqrt{3}$ and $2-\sqrt{3}$, $3\sqrt{3}+\sqrt{5}$ and $3\sqrt{3}-\sqrt{5}$ are conjugate surds respectively. The product to two conjugate surds is interesting to note . For example.

$$\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)$$

$= 2^2 - \left(\sqrt{3}\right)^2$, [according to $(a+b)(a-b) = a^2 - b^2$]

$= 4 - 3 = 1$

Similarly, $\left(3\sqrt{3}+\sqrt{5}\right)\left(3\sqrt{3}-\sqrt{5}\right)$

$$= \left(3\sqrt{3}\right)^2 - \left(\sqrt{5}\right)^2$$

$$= 27 - 5 = 22$$

Thus we find that the product of a bionomial surd and its conjugate is a rational number. This is used in solving various problems of surds.

Rationalisation : You have seen that the product of a binomial surd and its conjugate is a rational number.

Thus

$$\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right) = \left(\sqrt{3}\right)^2 - 1^2 = 3 - 1 = 2$$

You have also seen; $r(3) \times \sqrt{3} = 3$

$$2\sqrt{5} \times \sqrt{5} = 2 \times 5 = 10$$

This process of multiplying a surd by another to convert it into a rational number is called *rationalisation* and one of those two surds is called a *rationalising factor* of the other.

Thus, $\sqrt{3} - 1$ is the rationalising factor of $\sqrt{3} + 1$.
$\sqrt{3} + 1$ " " " $\sqrt{3} - 1$
$\sqrt{3}$ " " " $\sqrt{3}$
$\sqrt{5}$ " " " $2\sqrt{5}$

Remarks : (1) The rationalising factor of a binomial surd is its conjugate surd.

(2) If the denominator of a fraction be a surd, then the denominator of the fraction is made free of surd by multiplying both the numerator and denominator by the rationalising factor of that surd.

Thus ,

$$\frac{1}{\sqrt{3}} = \frac{1\times\sqrt{3}}{(\sqrt{3}\times\sqrt{3})} = \frac{\sqrt{3}}{3}$$

$$\frac{2\sqrt{2}}{\sqrt{5}} = \frac{2\sqrt{2}\times\sqrt{5}}{\sqrt{5}\times\sqrt{5}} = 2\sqrt{10}$$

Division of surds : To divide a surd by another, they are expressed in the form of a fraction by taking the dividend and the divisor as the numerator and denominator respectively. The result is obtained by rationalising the denominator and simplifying it. Thus,

$$\sqrt{2} \div \sqrt{6} = \frac{\sqrt{2}}{\sqrt{6}} = \frac{\sqrt{2} \times \sqrt{6}}{\sqrt{6} \times \sqrt{6}} = \frac{\sqrt{12}}{6} = \frac{2\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$$

$$\left(\sqrt{3} + 1\right) \div \left(\sqrt{3} - 1\right) = \frac{\sqrt{3} + 1}{\sqrt{3} - 1} = \frac{\sqrt{(3+1)^2}}{(\sqrt{3}-1\ (\sqrt{3}+1)}$$

$$= \frac{3 + 2\sqrt{3}+1}{3-1} = \frac{4 + 2\sqrt{3}}{2} = 2 + \sqrt{3}$$

Remark : A rational number can be divided by a surd in the same way. for example,

$$6 \div \sqrt{2} = \frac{6}{\sqrt{2}} = \frac{6\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$$

Example 1. Reduce $\sqrt{32}$ and $\sqrt{48}$ to their simplest forms

$$\sqrt{32} = \sqrt{2\times2\times2\times2\times2} = 2\times2\times\sqrt{2} = 4\sqrt{2}$$

$$\sqrt{48} = \sqrt{2\times2\times2\times2\times3} = 2\times2\times\sqrt{3} = 4\sqrt{3}$$

Example 2 . Simplify :

(i) $\sqrt{12} + \sqrt{18} + \sqrt{27} - \sqrt{32}$

(ii) $3\sqrt{2} - \sqrt{8} - \sqrt{3} + \sqrt{75}$

(i) $\sqrt{12} + \sqrt{18} + \sqrt{27} - \sqrt{32}$

$$\sqrt{2\times2\times3} + \sqrt{3\times3\times2} + \sqrt{3\times3\times3} - \sqrt{2\times2\times2\times2\times2}$$

$$= 2\sqrt{3} + 3\sqrt{2} + 3\sqrt{3} - 4\sqrt{2}$$

$$= \left(2\sqrt{3} + 3\sqrt{3}\right) + \left(3\sqrt{2} - 4\sqrt{2}\right)$$

$$= 5\sqrt{3} - \sqrt{2}$$

(*ii*) $3\sqrt{2}-\sqrt{8}-\sqrt{3}+\sqrt{75}$

$=3\sqrt{2}-2\sqrt{2}-\sqrt{3}+5\sqrt{3}$

$=\sqrt{2}+4\sqrt{3}$

Example 3. Simplify :

(i) $3\sqrt{5}\times2\sqrt{10}\times3\sqrt{2}$

(ii) $\left(2\sqrt{3}+3\sqrt{7}\right)\left(2\sqrt{7}-3\sqrt{3}\right)$

(*i*) $3\sqrt{5}\times2\sqrt{10}\times3\sqrt{2}$

$= \quad 3\times2\times3\times\sqrt{5}\times\sqrt{10}\times\sqrt{2}$

$= \quad 18\times\sqrt{5\times10\times2}=18\times\sqrt{100}=18\times10=180$

(*ii*) $\left(2\sqrt{3}+3\sqrt{7}\right)\left(2\sqrt{7}-3\sqrt{3}\right)$

$= \quad 4\sqrt{21}-18+42-9\sqrt{21}$

$= \quad 24-5\sqrt{21}$

Example 4. Rationalise the denominators :

(*i*) $\frac{\sqrt{5}-1}{\sqrt{5}+1}$,

(*ii*) $\frac{1}{3\sqrt{5}-2\sqrt{10}}$

(i) $\frac{\sqrt{5}-1}{\sqrt{5}+1}=\frac{(\sqrt{5}-1)(\sqrt{5}-1)}{(\sqrt{5}+1)(\sqrt{5}-1)}=\frac{(\sqrt{5}-1)^2}{5-1}$

$= \quad \frac{5-2\sqrt{5}+1}{4}=\frac{6-2\sqrt{5}}{4}=\frac{3-\sqrt{5}}{2}$

(*iii*) $\frac{1}{3\sqrt{5}-2\sqrt{10}}=\frac{3\sqrt{5}+2\sqrt{10}}{(3\sqrt{5}-2\sqrt{10})\,(3\sqrt{5}+2\sqrt{10})}$

$= \quad \frac{3\sqrt{5}+2\sqrt{10}}{(3\sqrt{5})^2-(2\sqrt{10})^2}$

$= \quad \frac{3\sqrt{5}-2\sqrt{10}}{45-40}=\frac{3\sqrt{5}+2\sqrt{10}}{5}$

Example 5. Simplify :

$$\frac{\sqrt{5}}{\sqrt{3}+\sqrt{2}}-\frac{3\sqrt{3}}{\sqrt{5}+\sqrt{2}}+\frac{2\sqrt{2}}{\sqrt{5}+\sqrt{3}}$$

$$\text{1st part} = \frac{\sqrt{5}}{\sqrt{3}+\sqrt{2}}=\frac{\sqrt{5}(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$$

$$= \frac{\sqrt{15}-\sqrt{10}}{3-2}=\sqrt{15}-\sqrt{10}$$

$$\text{2nd part} = \frac{3\sqrt{3}}{\sqrt{5}+\sqrt{2}}=\frac{3\sqrt{3}(\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})}$$

$$= \frac{3\sqrt{3}(\sqrt{5}-\sqrt{2})}{(\sqrt{5})^2-(\sqrt{2})^2}=\frac{3\sqrt{3}(\sqrt{5}-\sqrt{2})}{5-2}$$

$$= \frac{3\sqrt{3}(\sqrt{5}-\sqrt{2})}{3}=\sqrt{15}-\sqrt{6}$$

$$\text{3rd part} = \frac{2\sqrt{2}}{\sqrt{5}+\sqrt{2}}=\frac{2\sqrt{2}(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$$

$$= \frac{2\sqrt{3}(\sqrt{5}-\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}=\frac{2\sqrt{2}(\sqrt{5}-\sqrt{3})}{5-3}$$

$$= \frac{2\sqrt{2}(\sqrt{5}-\sqrt{2})}{2}=\sqrt{10}-\sqrt{6}$$

the given expression

$$= \left(\sqrt{15}-\sqrt{10}\right)-\left(\sqrt{15}-\sqrt{6}\right)+\left(\sqrt{10}-\sqrt{6}\right)$$

$$= \sqrt{15}-\sqrt{10}-\sqrt{15}+\sqrt{6}+\sqrt{10}-\sqrt{6} = 0$$

Example 6 : If $x = \frac{\sqrt{3}+1}{\sqrt{3}-1}$, $y = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

find the value of $x^2 - xy + y^2$

$$x+y = \frac{\sqrt{3}+1}{\sqrt{3}-1}+\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{(\sqrt{3}+1)^2+(\sqrt{3}+1)^2}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{2\{(\sqrt{3})^2+1\}}{3-1}$$

$$= \frac{8}{2} = 4$$

$$xy = \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}-1}{\sqrt{3}+1} = 1$$

$$\therefore x^2 - xy + y^2$$

$$= (x+y)^2 - 3xy$$

$$= 4^2 - 3.1$$

$$= 16 - 3 = 13$$

Exercise 27

1. Express as a complete surd :
   (*i*) $2\sqrt{3}$, (*ii*) $3\sqrt{2}$, (*iii*) $5\sqrt{7}$, (*iv*) $10\sqrt{2}$

2. Express in the simplest form :
   (*i*) $\sqrt{18}$, (*ii*) $\sqrt{75}$, (*iii*) $\sqrt{80}$, (*iv*) $\sqrt{150}$, (*v*) $\sqrt{180}$

3. Simplify :
   (*i*) $\sqrt{2} + \sqrt{8}$, (*ii*) $\sqrt{3} + \sqrt{12} - \sqrt{27}$
   (*iii*) $\sqrt{8} - \sqrt{12} + \sqrt{18} - \sqrt{48}$
   (*iv*) $\sqrt{45} + \sqrt{75} + \sqrt{108} + \sqrt{125}$
   (*v*) $\sqrt{24} - \sqrt{54} + \sqrt{48} - \sqrt{192}$
   (*vi*) $2\sqrt{3} - \sqrt{48} + \sqrt{75} - \sqrt{27}$

4. Simplify :
   (*i*) $\sqrt{2} \times 2\sqrt{2}$, (*ii*) $\sqrt{12} \times 2\sqrt{3}$, (*iii*) $\sqrt{7} \times \sqrt{14}$
   (*iv*) $\sqrt{5} \times \sqrt{15} \times \sqrt{3}$, (*v*) $(2 + \sqrt{3})(3 + \sqrt{5})$
   (*vi*) $(2 + \sqrt{2} + \sqrt{3})(3\sqrt{2} + \sqrt{3})$
   (*viii*) $(\sqrt{3} + 1)(5 + \sqrt{3})(\sqrt{3} - 1)(5 - \sqrt{3})$

5. Make the denominators free from surds (Rationalise the denominators)

(*i*) $\frac{1}{\sqrt{2}}$ (*ii*) $\frac{2}{\sqrt{3}}$ (*iii*) $\frac{2\sqrt{3}+3\sqrt{2}}{\sqrt{6}}$

(*iv*) $\frac{\sqrt{2}+1}{\sqrt{2}-1}$ (*v*) $\frac{5+\sqrt{2}}{2-\sqrt{2}}$

6. Divide the first by the second :

(*i*) $2+\sqrt{3}, \sqrt{3}-2$ (*ii*) $3\sqrt{2}+\sqrt{5}, \sqrt{2}+1$

(*iii*) $2\sqrt{3}-\sqrt{2}, 2-\sqrt{3}$

7. If $m+\frac{1}{m}=\sqrt{3}$, find the values of (i) $m^2+\frac{1}{m^2}$ and

(ii) $m^3+\frac{1}{m^3}$

8. Simplify :

(*i*) $\frac{3\sqrt{8}-2\sqrt{12}+\sqrt{20}}{3\sqrt{18}-2\sqrt{27}+\sqrt{12}}$ (*ii*) $\frac{x+\sqrt{x^2-1}}{x-\sqrt{x^2-1}}+\frac{x-\sqrt{x^2-1}}{x+\sqrt{x^2-1}}$

(*iii*) $\frac{4\sqrt{3}}{2-\sqrt{2}}-\frac{30}{4\sqrt{3}-\sqrt{18}}-\frac{\sqrt{18}}{3-2\sqrt{3}}$

(*iv*) $\frac{3\sqrt{7}}{\sqrt{5}+\sqrt{2}}-\frac{5\sqrt{5}}{\sqrt{2}+\sqrt{7}}+\frac{2\sqrt{2}}{\sqrt{7}+\sqrt{5}}$

(*v*) $\frac{\sqrt{2}(2+\sqrt{3})}{\sqrt{3}(\sqrt{3}+1)}-\frac{\sqrt{2}(2-\sqrt{3})}{\sqrt{3}(\sqrt{3}-1)}$

(*vi*) $\frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}}-\frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}+\frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$

9. If $a=\frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $b=\frac{\sqrt{5}-1}{\sqrt{5}+1}$, find the value of $\frac{a^2+ab+b^2}{a^2-ab+b^2}$

10. If $a=\sqrt{5}+2$, find the simplest numerical values of

(*i*) $a-\frac{1}{a}$, (*ii*) $a^2+\frac{1}{a^2}$ and (*iii*) $a^3-\frac{1}{a^3}$

ANSWERS

Algebra

Exercise 1

1. (*i*) $6x^2 + 19xy + 15y^2 + 47x + 78y + 15$
(*ii*) $20a - 14a^3 - 5a^2 + 46a - 40$
(*iii*) $a^8 + a^4 + 1$ (*iv*) $24m^5 - 46m^4 + 9m^3 + 13m^2 - 3m$
(*v*) $\frac{1}{12}x^3 + \frac{1}{72}xy^2 - \frac{1}{3}y^3$ 2.(i) $a^6 - b^6$ (ii) $x^{16} + x^8 + 1$
(*iii*) $a^2c - a^2b + ab^2 - b^2c + bc^2 - ac^2$
(*iv*) $p^{16} - q^{16}$ (v) $2b^2c^2 + 2c^2a^2 + 2a^2b^2 - a^4 - b^4 - c^4$

3. (*i*) $x^5 - x^3 + 2x^2 - 3x + 1$
(*ii*) $x^4 + \frac{9}{2}x^3y - 2x^2y^2 - \frac{9}{4}xy^3 + \frac{8}{4}y^4$

4. (*i*) $a - 2b - 3c$ (ii) $x^2 + 4a^2$ (*iii*) $x + 2y + 3z$
(*iv*) $6a^2 - 11ab + 4b^2$ (*v*) $p^2 + q^2 + 4r^2 - pq - 2qr - 2rq$
(*vi*) quotient = $p^4 + p^3 + p^2 + p + 2$, Remainder = 1
(*vii*) quotient = $-3x^2 + 2x + 1$, remainder = $2x + 2$
(*viii*) quotient = $2x^2 + 2x + 4$, remainder = $x + 2$
(*ix*) quotient = $m^2 + m + 1$, remainder = $-4m + 8$
(*x*) quotient = $y^2 - \frac{1}{2}y + \frac{1}{3}$, remainder = $\frac{1}{2}y - \frac{10}{9}$

5. (*i*) quotient $2l + 3m$, remainder = $96lm^2$
(*ii*) quotient = $x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$, remainder =
0

6. $3x^2 - 2x - 5$ 7. $3x^2 - 11x + 8$

(x) quotient = $y^2 - \frac{1}{2}y + \frac{1}{3}$, remainder = $8.21a^2 - 32ab +$
$12b^2\ 9p^2 - 4q^2$

Exercise 2

1. (i) $1 - 9a + 27a^2$ (ii) $8x^3 + 36x^2y + 54xy^2 + 27y^2$ (iii) $a^3 + b^3 + c^3 + 3(a + b)(b + c)(c + a)$ (*iv*) $1 - x^3 - y^3 + 3x^2(1 - y) + 3y^2(1 - x) + 6xy - 3x - 3y$ (*v*) 997002999 (*vi*) 1003003001

2. (i) $125x^3$ (ii) 125000 (iii) $2x + y + 3z$ (iv) $\frac{a}{b}$

3. (i) 3 (ii) $\frac{1}{6}$ (iii) 40 (iv) 27 (v) 125 (vi) 28 (vii) $\frac{n^3 - m}{3n}$ (viii) 35 (*ix*) 25 (x) 20, 96, 104, 1008, 992

4. (i) 23, 21, 110, 527 (ii) ± 7, 45, 2207 (iii) $\frac{74}{5}$, $\frac{284}{5}$, $\frac{5458}{25}$ (iv) $\frac{79}{3}$, 115

5. (i) $125x^3 + 300x^2y + 240xy^2 + 64y^3$ (ii) $x^3y^3 - 9x^2y^2z + 27xyz^2 - 27x^3$ (iii) $x^9 - 1$ (vi) $p^6q^6r^6 + s^6$ (v) $m^6 - n^6$ (vi) $a^8 - 390625b^8$ (vii) $p^6 - \frac{3p^4}{q^2} + \frac{3p^2}{3q^4} - \frac{1}{q^6}$ (viii) 13.2

Exercise 3

1. (i) $(a + b)(a - b)(a^2 + b^2)(a^4 + b^4)(a^8 + b^8)$

(ii) $(1-x)(1+x)(1+x^2)(1+x+x^2)(1-x+x^2)(1-x^2+x^4)$

(iii) $\left(\frac{x}{2} + \frac{y}{3}\right)\left(\frac{x}{2} - \frac{y}{3}\right)\left(\frac{x^2}{4} + \frac{y^2}{9}\right)$ (iv) $3(5a - 7b)(3b - a)$

(v) $\left(t - \frac{1}{2}\right)\left(t^2 + \frac{t}{2} + \frac{1}{4}\right)\left(t^6 + \frac{t^2}{8} + \frac{1}{64}\right)$

(vi) $A(R - r)(R^2 + Rr + r^2 + hR + hr)$

(vii) $\left(m + \frac{1}{m} - 2\right)$

(viii) $(2x - 3y)(2x - 3y + 1)$ (ix) $(3p - 4q)(3p - 4q + a)$, [read $90p^2$ in place of $9p^2$

(*x*) $\left(\frac{x}{4} - \frac{4}{x}\right)\left(\frac{x}{4} + \frac{4}{x} + 1\right)\left(\frac{x}{4} + \frac{4}{x} - 1\right)$

(*xi*) $(m + n)(m - n)(3n^2 - m^2)$ (*xii*) $(3p + q)(3p - q)(81p^4 + 9p^2q^2 + 9^4)$

(*xiii*) $7x^2(x + 2y)(x - 2y)(x^2 + 4y^2)$

(*xiv*) $(2a - b - 3c)(4x^2 + 19b^2 + 9c^2 + 14ab + 24bc + 6ca)$

(*xv*) $\left(m^2 + \frac{m}{2} + \frac{1}{4}\right)\left(m^2 - \frac{m}{2} + \frac{1}{4}\right)\left(m^4 - \frac{m^2}{4} + \frac{1}{16}\right)$

2. (i) $(a - 6b)(a - 9b)$ (ii) $(x + 9)(x - 3)$ (iii) $(x + 11y)(x - 10y)$ (iv) $(x + 16y)(x - 11y)$ (v) $(x - 27)(x + 18)$ (vi) $(5x - 2y)(x - 5y)$ (vii) $(2x + 1)(7x - 3)$ (viii) $3(a - 9)(3a - 4)$ (ix) $(a - 2p)(5 - 3p)$ (x) $(4m + 3)(1 - 3m)$ (xi) $(3 - x)(3 + 4x)$ (xii) $(2a + b - 7)(2a + b - 4)$ (xiii) $(7p + 5q)(3p + q)$ (xiv) $(x + y - 5)(2x + 2y + 5)$ (xv) $(p + a + 2)(p - a - 1)$

3. (i) $(a + 1)(a + 2)(a - 3)$ (ii) $(x + 2)(x + 2)(x - 4)$
   (iii) $(x + a + 2)(x - a + 1)$ (iv) $(x^2 + 2x - 5)(x^2 + 2x - 6)$
   (v) $(x + p)\left(x + \frac{1}{p}\right)$

Exercise 4

1. (i) $x + 1$ (ii) $a + x$ (iii) $a + 3b$ (iv) $2a + b - 2c$ (v) $p - 1$ (vi) $x(x + 5)$ (vii) $x - 1$ (viii) $x + 2$ (ix) $p + q - r$ (x) $2p + 3$ (xi) $m - 2$ (xii) $p - q$

2. (i) $x - 1$ (ii) $x^2 + 2x + 3$ (iii) $p^2 - p + 1$ (iv) $m^2 - 2m + 3$
   (v) $x^2 - 7x + 12$ (vi) $x^2 + xy + 2y^2$ (vii) $m - 1$

3. H.C.F. OR $A.B\,4(a^2 - b^2)$; H.C.F. of $B, C\,6(a - b)$; H.C.F. of $A, B, C - 2(a - b)$

Exercise 5

1. (i) $12(x + 1)^2(x^2 - x + 1)$ (ii) $x^3(x - 1)(x - 2)(x + 2)(x + 3)$
   (iii) $(y + z)(x + y + x)(x - y + z)(x + y - z)$
   (iv) $(2x + 1)(2x - 1)(4x^2 + 2x + 1)(4x^2 - 2x + 1)(4x^2 + 1)$
   (v) $(x + 3)(x + 4)(x + 5)(x^2 - 4x + 16)$
   (vi) $(x - 2)(x - 7)(x - 10)(x - 13)$
   (vii) $(y - 1)^2(y - 2)(y - 3)(y + 5)$
   (viii) $2n(1 + m)(1 - m)(2 + m)(1 + m + m^2)(1 - m + m^2)$
   (ix) $(p - 1)(p - 2)(p - 3)$

2. H.C.F. = $3x + 7$, L.C.M. = $(3x + 7)(5x - 3)(x - 10)$

3. H.C.F. = $x - 8$, L.C.M. = $(x - 8)(x + 7)(x - 12)$

4. product = $x^4 - 16x^2 + 25x^2 + 90x - 1411$

5. $x^3 - 2x^2 + 15x - 2$ 6. $2x^2 + 11x - 6$

Exercise 6

1. 0 2 $\dfrac{a+1}{a}$ 3. 1 4. a 5. 0 6. $p+q$

7. $\dfrac{1}{1+m}$ 8. $\dfrac{x+1}{x-3}$ 9. $\dfrac{x-y-z}{x+y+z}$ 10. 0 11. 1

Exercise 7

1. 11 2. 30 3. 9 4. 7

5. $\dfrac{p+q}{2}$ 6. $a+b$ 7. $\dfrac{1}{3}$ 8. $2\dfrac{3}{7}$

Exercise 8

1. $C = \dfrac{5F - 160}{9}, F = \dfrac{9C + 160}{5}$ 2. $f = \dfrac{v - u}{t}$ 3. $T = \dfrac{PV}{R}$

4. $r_1 = \dfrac{Rr_2}{r_2 - R}$ 5. $r = \dfrac{nE - RI}{nI}$ 6. $S = \dfrac{Pg}{g - P}$ 7. $f = \dfrac{uv}{u + v}$

Exercise 9

1. $\left.\begin{aligned} x + y &= 60 \\ x - y &= 10 \end{aligned}\right\}$ 2. $\left.\begin{aligned} 2x - 3y &= 15 \\ x - 2y &= 6 \end{aligned}\right\}$ 3. $\left.\begin{aligned} 2x - y &= 4 \\ 3x - y &= 3 \end{aligned}\right\}$

4. $\left.\begin{aligned} x + y &= 80 \\ 2x - 3y &= 20 \end{aligned}\right\}$ 5. $\left.\begin{aligned} 7x - 2y &= 0 \\ x - y &= 5 \end{aligned}\right\}$

6. $\left.\begin{aligned} 3x + 4y &= 80 \\ 27x - 44y &= 760 \end{aligned}\right\}$ cost of 1 kg. tea Rs. x
cost of 1 Kg. suger Rs. y

7. $\left.\begin{array}{l}(x+y)=30\\ x-y=4\end{array}\right\}$

8. $\left.\begin{array}{l}x=6\\ y=x+3\end{array}\right\}$

9. $\left.\begin{array}{l}x+y=10\\ x+3y=18\end{array}\right\}$ speed of $A = x$
speed of $B = y$

10. $\left.\begin{array}{l}y-2x=4\\ 3x-y=5\end{array}\right\}$ x = No. of boys
y = No. of mangoes

11. $\left.\begin{array}{l}x+y=6\\ x-y=4\end{array}\right\}$ speed of ore = x =
speedof current = y

12. $\left.\begin{array}{l}2x-3y+4=0\\ 5x-7y-8=0\end{array}\right\}$ x = present age of Swaswati
y = present age of Srabanti

13. $\left.\begin{array}{l}\dfrac{30}{x-y}+\dfrac{44}{x+y}=10\\ \dfrac{40}{x-y}+\dfrac{55}{x+y}=13\end{array}\right\}$ x = speed of boat
y = speed of current

14. $\left.\begin{array}{l}\dfrac{x}{y}-\dfrac{x}{y+10}=2\\ \dfrac{x}{y-15}-\dfrac{x}{y}=6\end{array}\right\}$ x = savings
y = No. of mangoes

15. $\left.\begin{array}{l}\dfrac{y}{x}-\dfrac{y}{x+2}=2\\ \dfrac{y}{x-2}-\dfrac{y}{x}=8\end{array}\right\}$ x = normal speed
y = distance

Exercise 10

1. (*i*) $\left.\begin{array}{l}x=2\\ y=3\end{array}\right\}$ (*ii*) $\left.\begin{array}{l}x=2\\ y=-1\end{array}\right\}$
(*iii*) No common solution (*iv*) No common solution

2. (*i*) $\left.\begin{array}{l}x=2\\ y=3\end{array}\right\}$ (*ii*) $\left.\begin{array}{l}x=2\\ y=-1\end{array}\right\}$
(*iii*) No common solution (*iv*) $\left.\begin{array}{l}x=-5\\ y=5\end{array}\right\}$

3. (*i*) $\left.\begin{array}{l}x=3\\ y=1\end{array}\right\}$ (*iv*) $\left.\begin{array}{l}x=-1\\ y=-1\end{array}\right\}$ (*iii*) $\left.\begin{array}{l}x=4\\ y=-1\end{array}\right\}$
(*iv*) $\left.\begin{array}{l}x=21\\ y=24\end{array}\right\}$ (*v*) $\left.\begin{array}{l}x=-2\\ y=3\end{array}\right\}$ (*vi*) $\left.\begin{array}{l}x=10\\ y=4\end{array}\right\}$

(vii) $\left.\begin{array}{l} x = 1 \\ y = 1 \end{array}\right\}$ (viii) $\left.\begin{array}{l} x = 2 \\ y = 3 \end{array}\right\}$ (ix) $\left.\begin{array}{l} x = 7 \\ y = 9 \end{array}\right\}$

(x) $\left.\begin{array}{l} x = 38 \\ y = 10 \end{array}\right\}$ (xi) $\left.\begin{array}{l} x = 6 \\ y = 5 \end{array}\right\}$ (xii) $\left.\begin{array}{l} x = 2 \\ y = 3 \end{array}\right\}$

(xiii) $\left.\begin{array}{l} x = 2 \\ y = \frac{2}{3} \end{array}\right\}$ (xiv) $\left.\begin{array}{l} x = 4\frac{10}{11} \\ y = 3\frac{8}{11} \end{array}\right\}$ (xv) $\left.\begin{array}{l} x = 12 \\ y = 8 \end{array}\right\}$

Exercise 11

I. (1) Possible, x = 1, y = 2 satisfy both the equations.

(2) Possible, x = 1, y = 3 satisfy both the equations.

(3) Not possible, no value of the chart satisfy both the equation.

II. 1. $\left.\begin{array}{l} x = 4 \\ y = 2 \end{array}\right\}$ 2. $\left.\begin{array}{l} x = 2 \\ y = -1 \end{array}\right\}$ 3. $\left.\begin{array}{l} x = 4 - \\ y = -2 \end{array}\right\}$

4. $\left.\begin{array}{l} x = 3 \\ y = 2 \end{array}\right\}$ 5. $\left.\begin{array}{l} x = -8 \\ y = 7 \end{array}\right\}$ 6. $\left.\begin{array}{l} x = 2 \\ y = \frac{1}{2} \end{array}\right\}$

7. $\left.\begin{array}{l} x = 9 \\ y = 1 \end{array}\right\}$ 8. $\left.\begin{array}{l} x = 7 \\ y = 4 \end{array}\right\}$ 9. $\left.\begin{array}{l} x = 4 \\ y = 10 \end{array}\right\}$

10. $\left.\begin{array}{l} x = 51 \\ y = 62 \end{array}\right\}$ 11. $\left.\begin{array}{l} x = 1 \\ y = 1 \end{array}\right\}$ 12. $\left.\begin{array}{l} x = \frac{6}{5} \\ y = \frac{5}{5} \end{array}\right\}$

13. $\left.\begin{array}{l} x = 8 \\ y = 5 \end{array}\right\}$ 14. $\left.\begin{array}{l} x = 7 \\ y = 9 \end{array}\right\}$

Exercise 12

1)

x	3	2	1	0	5	6	7	8
y	1	2	3	4	–1	–2	–3	–4

2)

x	3	5	7	9	–1	–3	–5	–7
y	1	2	3	4	–1	–2	–3	–4

3)

x	3	$\frac{7}{2}$	4	$\frac{9}{2}$	2	$\frac{3}{2}$	1	$\frac{1}{2}$
y	1	2	3	4	–1	–2	–3	–4

4)

x	7	$\frac{11}{2}$	4	$\frac{5}{2}$	10	$\frac{23}{2}$	13	$\frac{29}{2}$
y	1	2	3	4	–1	–2	–3	–4

IV. $\left.\begin{array}{l} 5x + 7y = 41, \\ x + y = 7 \end{array}\right\}$

x	$\frac{34}{5}$	$\frac{27}{5}$	4	$\frac{18}{5}$	$\frac{48}{5}$	11	$\frac{62}{5}$	$\frac{69}{5}$
y	1	2	3	4	–1	–2	–3	–4

x	6	5	4	3	8	9	10	11
y	1	2	3	4	–1	–2	–3	–4

Exercise 13

1. $\left.\begin{array}{l} x = 3 \\ y = 1 \end{array}\right\}$
2. $\left.\begin{array}{l} x = 1 \\ y = 2 \end{array}\right\}$
3. $\left.\begin{array}{l} x = 7 \\ y = 6 \end{array}\right\}$
4. $\left.\begin{array}{l} x = 36 \\ y = 12 \end{array}\right\}$
5. $\left.\begin{array}{l} x = 12 \\ y = 6 \end{array}\right\}$
6. $\left.\begin{array}{l} x = 21 \\ y = 24 \end{array}\right\}$
7. $\left.\begin{array}{l} x = 2 \\ y = 2 \end{array}\right\}$
8. $\left.\begin{array}{l} x = 2 \\ y = -1 \end{array}\right\}$
9. $\left.\begin{array}{l} x = 3 \\ y = -4 \end{array}\right\}$
10. $\left.\begin{array}{l} x = 6 \\ y = 8 \end{array}\right\}$
11. $\left.\begin{array}{l} x = 3 \\ y = 2 \end{array}\right\}$
12. $\left.\begin{array}{l} x = 2 \\ y = 1 \end{array}\right\}$
13. $\left.\begin{array}{l} x = 7 \\ y = 9 \end{array}\right\}$
14. $\left.\begin{array}{l} x = -2 \\ y = 3 \end{array}\right\}$
15. $\left.\begin{array}{l} x = \frac{6}{5} \\ y = \frac{6}{5} \end{array}\right\}$
16. $\left.\begin{array}{l} x = \frac{1}{4} \\ y = 6 \end{array}\right\}$
17. $\left.\begin{array}{l} x = 7 \\ y = 4 \end{array}\right\}$
18. $\left.\begin{array}{l} x = 2 \\ y = \frac{1}{2} \end{array}\right\}$
19. $\left.\begin{array}{l} x = \dfrac{c(b-c)}{a(b-a)} \\ y = \dfrac{c(c-a)}{b(b-a)} \end{array}\right\}$
20. $\left.\begin{array}{l} x = a \\ y = b \end{array}\right\}$

Exercise 14

1. $\left.\begin{array}{l} x=2 \\ y=1 \end{array}\right\}$ 2. $\left.\begin{array}{l} x=1 \\ y=2 \end{array}\right\}$ 3. $\left.\begin{array}{l} y=23. \\ y=-\frac{1}{4} \end{array}\right\}$

4. $\left.\begin{array}{l} x=3 \\ y=-1 \end{array}\right\}$ 5. $\left.\begin{array}{l} x=3 \\ y=2 \end{array}\right\}$ 6. $\left.\begin{array}{l} y=4 \\ y=-1 \end{array}\right\}$

7. $\left.\begin{array}{l} x=16 \\ y=-4 \end{array}\right\}$ 8. $\left.\begin{array}{l} x=3 \\ y=-\frac{9}{5} \end{array}\right\}$ 9. $\left.\begin{array}{l} y=5 \\ y=9 \end{array}\right\}$

10. $\left.\begin{array}{l} x=1 \\ y=2 \end{array}\right\}$ 11. $\left.\begin{array}{l} x=16 \\ y=4 \end{array}\right\}$ 12. $\left.\begin{array}{l} y=a+b \\ y=b-a \end{array}\right\}$

13. $\left.\begin{array}{l} x=a+b \\ y=b-a \end{array}\right\}$ 14. $\left.\begin{array}{l} x=21 \\ y=24 \end{array}\right\}$ 15. $\left.\begin{array}{l} y=3 \\ y=2 \end{array}\right\}$

16. $\left.\begin{array}{l} x=\frac{c(c-b)}{a(a-b)} \\ y=\frac{c(c-a)}{b(b-a)} \end{array}\right\}$ 17. $\left.\begin{array}{l} x=a \\ y=b \end{array}\right\}$ 18. $\left.\begin{array}{l} x=\frac{ac-bc-b}{a^2-b^2} \\ y=\frac{ac-bc+a}{a^2-b^2} \end{array}\right\}$

19. $\left.\begin{array}{l} x=\frac{a}{a^2+b^2} \\ y=\frac{b}{a^2+b^2} \end{array}\right\}$ 20. $\left.\begin{array}{l} x=a \\ y=b \end{array}\right\}$

Exercise 15

1. Rs. 425, Rs. 752. Rs. 3.80 3.u = 4, f = 24. Daben 12 days, Ramen 9 days 5. Pradip 40 Kg., 6, 40 years 20 years 7. five rupee note 22, ten rupee note48 8. 1st solution $77\frac{7}{9}$., 2nd solution $72\frac{2}{9}$ gr. 9. 1st portion Rs. 6,000 2nd portion Rs. 5,000 10. 1st number $9\frac{2}{3}$, 2nd number $15\frac{1}{3}$, $11.\frac{5}{7}$, 12. Rs. 16, Rs. 28, 13. Akhil babu 235, Amar babu 160 14. Latif Rs. 160, Ishan Rs, 120 15. 63, 16. 35, 17. length 15m., breadth = 12 m. 18.25 miles/hour, 5miles/hour 19. Rs. 32, Rs. 15 20. 36 years, 27 years 21. 40, 25. 22. bi-cycle Rs. 1,200

and watch Rs. 500 23. Rs. 24, 13 boys 24. 15 boxes, 1200 oranges, 25. 30 hours and 20 hours 26. No. of rupee 180, No. of 50 paise coin 160 27. 540 km, 36 km.hour 28. 8 km./hour and 3 km./hour. 29. 96 and 24 30. 100 km. 25 km./hour.

Exercise 16

1. *(i)* On the positive sude if x axis *(ii)* on the positive side of y SXOS *(iii)* on the negative side of x axis *(iv)* on the negative side of y axis *(iv)* on the negative side of y axis *(v)* 1st quadrant *(vi)* 2nd quadrant *(vii)* 3rd quadrant *(viii)* 4th quadrant 20. (2,2) 21. $(4,_1)$ 22. (0,5) 23. point of intersection (2,0) 24. point of intersection (7,1) 27. 5 units 28. 28 sq. units.

Exercise 17

1. 35 : 64 2. 4 : 33 3. 10 : 17 4. $ace : bdf$ 5. $xy : z^2$ 6. $\frac{9}{10}$ 7. $\frac{7}{5}$ 8. 40 : 19 9. 9 : 4 10. $-\frac{11}{133}$ 11. $\frac{247}{1002}$ 12. $\frac{8}{5}$ 13. 2 14. $\frac{bx-ay}{a-b}$ 15. $\frac{mv-np}{m-n}$

Exercise 18

1. (i) 8 *(ii)* 15 (iii) $\frac{bc}{a}$ (iv) $\frac{c^2d}{a}$ 2.(i) 18 (ii) $\frac{b^2}{a}$ *(iii)* $\frac{y^2z^2}{x^3}$ 3. (i) ± 8 (ii) $\pm\frac{1}{9}$ (iii) ab (iv) x^2y (v) x^2-y^2 4. 3 5. 162 6. 3 7. $\frac{bc-ad}{a-b-c+d}$.

Exercise 19

7. 0, 10. $\frac{4a}{a^2+4}$

Miscellaneous Exercise 1

1. (*i*) $(x^2+2xy+2y^2)(x^2-2xy+2x^2)$ (*ii*) $(2x^2+6x+9)$ $(2x^2-6x+9)$ (*iii*) $(a^2-2)(a^2+2)(a^2+2a+2)(a^2-2a+2)$ (*iv*) $(x^2+xy+y^2)(x^2-xy+y^2)$ (*v*) $(x^2+3x+1)(x^2-3x+1)$ (*vi*) $(p^2+3pq-q^2)$ (*vii*) $(m^2-m+1)(m^2+m+1)$ (m^4-m^2+1) (*viii*) $(a-b+c)(a-b-c)$ (*ix*) $(b+c-a)(b-c+a)$ (*x*) $(3a-2b)(3a+2b+2c)$ (*xi*) $(x-y)(x+y-1)$ (*xii*) $(a-2c)$ $(a-6b+2c)$ (*xiii*) $(3a+b+c)(a+b-c)$ (*xiv*) $(a+b-c)(a-3b+c)$ (*xv*) $(a+3b-2c-5d)(a-3b-2c+5d)$

2. (*i*) $(x^2+3a^2)(x^4-3a^2x^2+9a^4)$ (*ii*) $a(2a-5b)(4a^2+10ab+25b^2)$

(*iii*) $(2x+1)(4x^2-38x+127)$ (*iv*) $(m-3n)(m^2+3n^2)$

(*v*) $(m-n)(m+n)(m^2+mn+n^2)(m^2-mn+n^2)$

(*vi*) $(1-x)(1+x)(1+x^2)(1+x+x^2)(1-x+x^2)$ $(1-x^2x^4)$

(*vii*) $(a-2)(a^2+2a-2)$ (*viii*) $(a+2)(a+3(a-5)$

(*ix*) $(a-3)(a^2+2a+6)$ (*x*) $(a-1)(a^2-a-1)$

3. (*i*) $(a+2)(a+3)$ (*ii*) $(a+6)(a-1)$ (*iii*) $2a(a+3)(a+9)$
(*iv*) $3(2a-1)$ (*v*) $(a-b-3)(a-b+2)$ (*vi*) $(a+2b)(a-9b+1)$
(*vii*) $(x+1)(x+2)(x-3)(x-4)$ (*viii*) $(x-1)(x+1)(x^2-2)$
(*ix*) $(x^2+5x+1)(x^2+3x+1)$ (*x*) $(x-1)^2(x^2-2x-4)$

Miscellaneous Exercise 2

1. (*i*) $x-2$ (*ii*) $x-1$ (*iii*) $a-b+c$ (*iv*) $a+b-c$ (*v*) $a+b$ (*vi*) *1* (*vii*) $x-2$ (*viii*) $ab(a-b)$ (*ix*) $a(a-2)$ (*x*) $a(2x-1)$ (*xi*) x^2+1

2. (*i*) x^2-5x+3 (*ii*) x^2-3x-4 (*iii*) $x+2$ (*iv*) $2x(x^2+x+1)$ (*v*) $x3-3x^2+3x-1$ (*vi*) $2(x-2)$ (*vii*) $x+3$ (*viii*) $x-3$ (*ix*) $2x(x^2+x-1)$ (*x*) $x-1$

3. (*i*) $(x-1)(x-2)(x-3)$ (*ii*) $x^2(x+2)(x-3)(x+5)$ (*iii*) $(x+2)(2x-1)(3x+1)$ (*iv*) $x^2(x+2)(x-2)(x+4)$ (*v*) $(a+b)$ $(b+c)(c+a)$ (*vi*) $6a(x+2)(x+1)^2$ (*vii*) $6(x+2)(x-2)^2$ (*viii*) $(a+b-c)(b+c-a)(c+a-b)$ (*ix*) $(x-1)(x+1)(x^2-x+1)(x^2+x+1)$ (*x*) $ax(x-a)(x+a)$

Miscellaneous Exercise 3

1. (*i*) $x = 1, y = 1$ (*ii*) $x = \frac{3}{2}, y = 2$ (*iii*) $x = 7, y = 13$ (*iv*) $x = 6, y = 2$ (*v*) $x = \frac{1}{3}, y = \frac{1}{5}$ (*vi*) $x = 4, y = 4$ (*vii*) $x = \frac{cq - br}{aq - bp}$ $y = \frac{ar - cp}{aq - bp}$

2. (*i*) $x = 2, y = 1$ (*ii*) $x = 1, y = -1$ (*iii*) $x = 3, y = -2$ (*iv*) $x = 3, y = 4$ (*v*) $x = 5, y = 7quad$ (*vi*) $x = 4, y = 5$ (*vii*) $x = 0, y = 3$

3. (*i*) $x = \frac{1}{2}(a + \mathrm{b}), y = \frac{1}{2}(a - \mathrm{b})$, (*ii*) $x = 3, y = 2$ (*iii*) $x = \frac{1}{3}$, $y = \frac{1}{2}$ (*iv*) $x = 1, y = \frac{1}{3}$ (*v*) $x = 4, y = 6$ (*vi*) $x = \frac{18}{7}, y = \frac{8}{3}$ (*vii*) $x = \frac{18}{7}, y = \frac{8}{3}$ (*vii*) $x = -4, y = \frac{4}{3}$ (*viii*) $x = 2, y = 1$ (*ix*) $x = 8$, $y = 5$ (*x*) $x = q + p, y = p - q$

4. (*i*) $p = \frac{1}{2}, q = \frac{1}{3}$ (*ii*) $x = 4, y = -1$ (*iii*) $x = 1, y = 1$ (*vi*) $m = \frac{4}{3}, n = 1$ (*v*) $x = 10, y = 5$ (*vi*) $x = 6, y = -2$ (*vii*) $x = \frac{11}{12}, y = \frac{12}{19}$ (*viii*) $x = 16, y = 4$ (*ix*) $x = x = 4, y = 6$ (*x*) $x = a, y = b$ (*xi*) $x = \frac{a}{a^2 + b^2}$ $y = \frac{b}{a^2 + b^2}$

5. (*i*) $x = 4, y = 1$ (*ii*) $x = \frac{82}{29}, y = \frac{98}{29}$ (*iii*) $x = 3, y = 4$ (*iv*) $l = 2, m = 4$ (*v*) $u = 5, v = 3$ (*vi*) $x = 5, y = 1$ (*vii*) $x = \frac{6}{5}, y = \frac{6}{5}$ (*viii*) $x = 3, y = -1$ (*ix*) $x = 2, y = -2$ (*x*) $x = 1, y = 1$ (*xi*) $x = \frac{a + b}{a - b}$ $y = \frac{a - b}{a + b}$

6. (*i*) $x = \frac{5}{2}, y = 2$ (*ii*) $x = 7, y = 8$ (*iii*) $x = -2, y = 3$ (*iv*) $x = 2, y = \frac{1}{5}$ (*v*) $x = 4, y = \frac{1}{4}$ (*vi*) $x = \frac{7}{26}, y = \frac{7}{9}$, (*vii*) $x = 1, y = 1$ (*viii*) $x = \frac{1}{7}, y = \frac{1}{9}$ (*ix*) $x = -\frac{1}{2}, y = \frac{1}{2}$ (*x*) $x = \frac{3}{5}, y = \frac{3}{7}$ (*xi*) $x = 2, y = \frac{3}{8}$ (*xii*) $x = 1, y = 1$ (*xiii*) $x = 6, y = 3$ (*xiv*) $x = \frac{1}{4}$, $y = \frac{1}{7}$ (*xv*) $x = 2, y = 1$ (*xvi*) $x = 2, y = 1$ (*xvii*) $x = 2, y = 1$ (*xviii*) $x = 2, \quad y = -2$

7. $x = 2, y = 1$; will be $u = 2, v = 1$

8. (*i*) Only one solution (*ii*) No solution (*iii*) Only one Solution (*iv*) Infinite number of solutions (*v*) No solution.

Miscellaneous Excercise 4

1. $x + y = \frac{1}{7}(10x + y)$, Number $= 42$

2. $x - 1 = y + 1$, Number $= 75$

$$\frac{10y + x}{10x + y} = \frac{19}{25}$$

$$\left\{\begin{array}{l} x - \text{digit in the ten's place} \\ y - \text{digit in the unit's place} \end{array}\right\}$$

3. $10x + y = 5(x + y)$, Number $= 45$

$$(10x + y) + 9 = 10y + x$$

x – digit in the ten's place }
y – digit in the unit's place

4. $\frac{x}{y + 1} = \frac{1}{3}$, Fraction $= \frac{3}{8}$ (x–numerati or, y – denominator)

$$\frac{x + 1}{y} = \frac{1}{2}$$

5. $\frac{y - x}{x + y} = \frac{1}{6}$, Fraction $= 5$

$$x - 1 = \frac{1}{2}(y + 1)$$

(x – numerator, y – denominator)

6. $x + y = 7$,
$x + 1 = y$ Fraction $= \frac{3}{4}$

(x – numerator, y – denominator)

7. $x = 2y$, Age of father $= 56$yrs.

$x - 14 = 3(y - 14)$ Age of son $= 28$yrs

$$\left\{\begin{array}{l} x\,yrs. - \text{Age of father} \\ y\,Yrs. - \text{Age of Son} \end{array}\right\}$$

8. $5x + 4y = 28,$ Rs. 6

$2x + 5y = 28$

$$\left\{\begin{array}{l}\text{Rs. } x - \text{cost of 1 litre of milk}\\ \text{Rs. } y - \text{cost of 1 litre of kerosine}\end{array}\right\}$$

9. $5x + 2y = 9,$ $Rs.$ 1

$4x + 3y = 10$

$$\left.\begin{array}{l}\text{Rs.}x - \text{price of one pencil}\\ \text{Rs.}y - \text{price of one pen}\end{array}\right\}$$

10. $8x + 10y = 50,$ Rs. 5.50

$5x + 6y = 30.50$

$$\left.\begin{array}{l}\text{Rs.}x - \text{price of one quire of paper}\\ \text{Rs.}y - \text{price of one bound exercise book}\end{array}\right\}$$

11. $x + 2 = 2(y - 2),$ Kajari – Rs. 10.00

$y + 1 = x - 1$ Sarbari – Rs. 8.00

$$\left.\begin{array}{l}\text{Rs.}x - \text{Kajaris' amount}\\ \text{Rs.}y - \text{Sarbari's amount}\end{array}\right\}$$

12. $y + 30 = 2(x - 30),$ *Erfan – 34*

$x + 10 = 3(y - 10)$ *Liakat – 62*

$$\left\{\begin{array}{l}x = \text{No. of mangoes of Liakat}\\ y = \text{No. of mangoes of Erfan}\end{array}\right\}$$

13. $\frac{12}{x} + \frac{12}{y} = 1,$ Suresh takes 20 days to do the work alone

$\frac{16}{x} + \frac{6}{y} = 1$

$$\left\{\begin{array}{l}\text{Suresh takes x days}\\ \text{and Ramesh y days}\\ \text{to do the work}\end{array}\right\}$$

14. $16x + 15y = 125,$ First man – $Rs.$ 5

$4x = 7y - 1$ Second man – $Rs.$ 3

$$\left\{\begin{array}{l}\text{Wage of first man Rs. } x\\ \text{Wage of second man = Rs.}y\end{array}\right\}$$

15. $9x - 9y = 27$ Speed of A = 6 km/hr.

$3x + 3y = 27$ Speed of B = 3 km/hr.

$\left\{ \begin{array}{l} \text{Speed of A} = x \text{ km/hr.} \\ \text{Speed of B} = y\text{km/hr.} \end{array} \right\}$

16. $\dfrac{x}{y+6} = \dfrac{x}{y} - 6,$ Distance = *720km*

$\dfrac{x}{y-4} = \dfrac{x}{y} + 6,$ Speed = *24km/hr.*

$\left\{ \begin{array}{l} \text{Distance} = x\text{km.} \\ \text{Speed} = y\text{km/hr.} \end{array} \right\}$

17. $\dfrac{24}{x+y} + \dfrac{12}{x-y} \quad 10$ Speed of boat in still water = 4 km/hr.

$\dfrac{36}{x+y} + \dfrac{16}{x-y} = 14$

$\left\{ \begin{array}{c} \text{Speed of boat in still water} = \text{x km/hr.} \\ \text{Speed of current} = \text{y km/hr.} \end{array} \right\}$

18. $\dfrac{33}{x+y} + \dfrac{55}{x-y} = 14,$ Rate or rowing = 8 km./hr.

\f(55,x+y) + \f(22,x− *Speed of current = 3km./hr.*

y) = $9\,\dfrac{24}{60}$

$\left\{ \begin{array}{l} \text{Rate of rowing} = x \text{ km/hr.} \\ \text{Speed of current} = y \text{ km/hr.} \end{array} \right\}$

19. $\dfrac{2x}{5} + \dfrac{5y}{9} = \dfrac{3x}{5} + \dfrac{4y}{9}$; Required ratio = 5 : 9

$\left\{ \begin{array}{l} \text{First mixture} - \text{x} \\ \text{Second mixture} - y \end{array} \right\}$

20. $\dfrac{y-50}{x-y-50} = \dfrac{5}{3},$

$\dfrac{y+25}{x-y-25} = \dfrac{11}{3}$ *Rs. 466.67 (approx.)*

$\left\{ \begin{array}{l} \text{monthly income} = \text{Rs. } x \\ \text{monthly expenditure} = \text{Rs.} y \end{array} \right\}$

21. $x + y = 990$, 540 students and

$\frac{95x}{100} = \frac{114y}{100}$ 450 students

$$\left\{ \begin{array}{l} \text{First} - x \\ \text{Second} - y \end{array} \right\}$$

22. $3x + 2y = 7800$, Moped – Rs. 3000 Cycle – Rs. x

$\frac{33x}{10} + \frac{5y}{2} = 9480$ Cycle – Rs. 600 Moped –Rs. y

23. $(x + 5)(y - 3) = xy - 35$, Length = 40m

$x - 10 = y + 10$ Breadth = 20m

$$\left\{ \begin{array}{l} Length = xm \\ Breadth = ym \end{array} \right\}$$

24. $(x - 2)(y + 3) = xy + 24$, *Breadth = 21m*

$(x + 2)(y - 2) = xy - 16$ *quad Length = 18m*

$$\left\{ \begin{array}{l} Length = xm \\ Breadth = ym \end{array} \right\}$$

25. $2\wedge(x + y) = 175$ Radius of garden = 11m

$x - y = 6$

$$\left\{ \begin{array}{l} Outerradius = xm \\ radius\ of\ garden = ym \end{array} \right\}$$

ANSWERS

Miscellaneous Exercise 5

1. *(i)* on the x-axis in the negative direction *(ii)* on the x-axis in the positive direction *(iii)* on the y-axis in the negative direction *(iv)* in the first quadrant *(v)* in the second quadrant *(vi)* in the fourth quadrant *(vii)* in the third quadrant *(viii)* on the y-axis in the positive direction 3. *(i)* lies *(ii)* does not lie *(iii)* lies *(iv)* does not lie 5. *(i)* 0,

7 *(ii)* 1, –1 *(iii)* 3. 6. *(i)* x = –2 *(ii)* x = 3 *(iii)* x = –6 *(iv)* x = 8 7. *(i)* –4, 11 *(ii)* 4 and 3 units *(iii)* 48 sq. units *(iv)* $7\frac{1}{2}$ sq. units *(v)* 30 sq. units 13 units *(vi)* (2,3), (7,3), (4,7) ; Area = 10 sq. units *(vii)* (3,0) (–3,7), (–3, –4); Area = 33sq. units *(viii)* 6 sq. units 8. *(i)* (5,1), *(ii)* (1,–3), *(iii)* (–1,–4) *(iv)* (–5,4) *(v)* (3,–4) *(vi)* (2,–1) 9. *(i)* x = 5, y = 1, *(ii)* x = 12.5, y = –5 *(iii)* x = 3, y = 1 *(iv)* x=–4, y = –2 *(v)* x = 1, y = 2 *(vi)* x = 4, y = –2 *(vii)* x = –1, y = 1 *(viii)* x = –7, y = –5 10. Two graphs are parallel to each other. 11. 90° 12. Graphs of two equations represent the same straight line. 13. $\frac{1}{2}, \frac{7}{2}, \frac{13}{2}$; No.

Miscellaneous Exercise 6

1. $a = 6, b = 9, c = 12$ 2. 2 : 1 5. (i) $ad : bc = 1 : 1, a + c : b + d = 5 : 3$

20. $x = 10, y = 4$

Esercise 21

1. Rs. 700 **2.** Rs. 15.20, 15 litres **3.** *(i)* 37.75 degrees *(ii)* 38.5 degrees **4.** 2 km, 5 hrs. **5.** Rs. 5.6, 25 lemons **6.** 45 thousands, 60 thousands **7.** After 4 hrs at a distance of 16 km from Sagarpur **8.** At 5 A.M., at a distance of 35 Km **9.** At 9-40 P.M., at a distance of 9 km from Ramnagar.

Exercise 22

1. $x^2 = 42 - x$, or, $x = (42 - x)^2$ 2. $x(x + 2) = 143$, *or*, $(x - 2)x = 143$

3. $x^2 + (x + 1^2 = 313$, *or*, $(x - 1)^2 + x^2 = 313$

4. $(x + 4)\left(\frac{80}{x} - 1\right) = 80x$

5. $x^2 + (x + 3)^2 = 15^2$, or, $x^2 + (x - 3)^2 = 15^2$

6. $\frac{300}{x} - \frac{300}{x+5} = 2$

7. $336 - x = \frac{x^2}{100}$ 8. $\frac{14}{x+2} + \frac{14}{x-2} = 5$

9. $\frac{2}{x} + \frac{2}{x+3} = 1$ or, $\frac{2}{x-3} + \frac{2}{xz} = 1$

10. $x(x + 6) = (10x + x + 6) - 12$ or, $(x - 6)x = 10(x - 6) + x - 12$

11. $(45 + 2x)(40 + 2x) - 45 \times 40 = 450$ 12. $\frac{18}{x+4} + \frac{6}{x-2} = 3$

[The forms of the equations depend on which one is taken to be unknown and what is that unknown assumed. Therefore one may get equations in other forms instead of forms given in the answer.]

Exercise 23

1. ± 2 **2.** ± 10 **3.** ± 5 **4.** ± 6 **5.** ± 3 **6.** $\pm\sqrt{\frac{3}{2}}$ 6.3 **7.** ± 2 **8.** ± 6 **9.** ± 9 **10.** $-6, 22$ **11.** $\pm\sqrt{ab}^2$ **12.** ± 1 **13.** $\frac{2}{3}, --\frac{5}{2}$ **14.** $\frac{4}{3}, -\frac{5}{14}$ **15.** $2, -\frac{6}{5}$ **16.** $7, -\frac{77}{17}$ **17.** $0, \frac{2}{3}$ **18.** $3, -4,$ **19.** $3, -\frac{68}{9}$ **20.** $0, -7$ **21.** $-a, -b$ **22.** 6, 9 **23.** $-25, 26$ **24.** 0, 1 **25.** 3a, 2a **26.** a, $-(a+b)$ **27.** $1, \frac{1}{3}$ **28.** $-2, 7$ **29.** $0, \frac{2ab-bc-ac}{a+b-2c}$.

Exercise 23

1. 9, 6 **2.** Length = 46 m **3.** 15 m **4.** 3 **5.** 7.5 m 20 km./hr. **7.** Length = 50 m, breadth = 40m **8.** 35 km/hr. **9.** 20 min. and 25 min. **10.** 15 km/hr. **11.** in 6 days **12.** 14 paise.

Exercise 24

6.

x	1	2
y	2	2

7.

x	1	1
y	1	2

8. { (1,1), (1,2), (1,3) (2,1)}

9. {(0,1), (0,2), (1,0), (1,1), (2,0)}

Exercise 27

1. (i) $\sqrt{12}$ (ii) $\sqrt{18}$ (iii) $\sqrt{175}$ (iv) $\sqrt{200}$ 2. (i) $3\sqrt{2}$ (ii) $5\sqrt{3}$ (iii) $4\sqrt{5}$ (iv) $5\sqrt{6}$ (v) $6\sqrt{5}$ 3. (i) $3\sqrt{2}$ (ii) 0 (iii) $5\sqrt{2} - 6\sqrt{3}$ (iv) $8\sqrt{5} + 11\sqrt{3}$ (v) $-\sqrt{6} - 4\sqrt{3}$ (vi) 0 4. (i) 4 (ii) 12 (iii) $7\sqrt{2}$ (iv) 15 (v) $6 + 3\sqrt{3} + 2\sqrt{5} + \sqrt{15}$ (vi) $9 + 6\sqrt{2} + 2\sqrt{3} + 4\sqrt{6}$ (vii) 44 5. (i)

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